Option Hedging using Explainable Artificial Intelligence (X Hedging)

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Bjørn André Aaslund[1], Johannes Berge[1], Ying Ni[2], Rita Pimentel[1]

[1] Norwegian University of Science and Technology
[2] Division of Mathematics and Physics, Mälardalen University, Sweden

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AI in option hedging

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Option Hedging

- Seller (writer) of an option has obligation if the option buyer (holder) exercise.
- Option sellers need to offset this risk.
- *Hedging* = *risk reduction*: buying or selling a certain amount of the opposite position via a *hedging instrument*.
- For market-makers: reduce risk by hedging, profit from bid–ask spread.

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AI in Finance- Machine learning (ML)

" Econometrics might be good enough to succeed in financial academia (for now), but succeeding in practice requires ML "

— Marcos López de Prado (2018)

López de Prado, Marcos (2018). Advances in Financial Machine Learning. Hoboken, NJ: John Wiley & Sons.

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EU: The Ethics Guidelines for Trustworthy AI

Figure reprinted from Ethics Guidelines for Trustworthy AI.

State-of-art

- Deep Hedging (neural networks): produce hedging strategies for any environment, but lacks explainability.
- Explainability within AI (XAI): to meet new demands of guidelines and regulations (Prenio and Yong 2021).
- Explainability and transparency: an important factor in AI principles by OECD, G20, EU, Germany, Hongkong, Singapore and US.
- Deep Hedging: does not achieve *local explainability* (explainability of individual decisions within a model).

Prenio, J. & Yong, J. (2021). Humans keeping AI in check–emerging regulatory expectations in the financial sector. Bank for International Settlements.

X Hedging: An Explainable Artificial Intelligence Hedging Framework

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Hedging positions

- Hedge against: a liability Z with maturity T (in our example European call option).
- Hedging occurs at discrete time steps $t_o = 0, t_1, \ldots, t_k, \ldots, t_n = T$.
- Hedging by: long $\delta_k \in \mathbb{R}$ units of hedging instrument S_k at time t_k.

Hedging positions:

$$(\delta \cdot S)_T := \sum_{k=0}^{n-1} \delta_k \cdot (S_{k+1} - S_k),$$

S_k: price of the underlying asset at time t_k.

Market frictions

- can account for market frictions such as transaction costs and liquidity constraints.
- Total sum of the market frictions is:

$$C_T(\delta) := \sum_{k=0}^{n} c_k(\delta_k - \delta_{k-1}).$$

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Fixed and proportional transaction costs

- Fixed transaction costs:

$$c_k(x) := \kappa \mathbb{1}_{|x| \geq \varepsilon},$$

  - cost constant: $\kappa > 0$.
  - hedging strategy change threshold: ε.
  - Indicator function for threshold: $\mathbb{1}$.

- Proportional transaction costs:

$$c_k(x) := \kappa S_k |x|.$$

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Final profit and loss (P&L)

Final P&L is given by the total portfolio position of the hedger at time step T

$$\text{P\&L}_T(Z,p,\delta) := -Z + p + (\delta \cdot S)_T - C_T(\delta),$$

- p: initial cash injection (option premium received by the hedger);
- Z: payoff of the liability is Z;
- If $\text{P\&L}_T < 0$: losses;
- Goal of the hedger: minimize the expected value of a loss function ℓ associated with the P&L, i.e.

$$\pi := \inf_{\delta} \mathbb{E}[\ell(\text{P\&L}_T(Z,p,\delta))].$$

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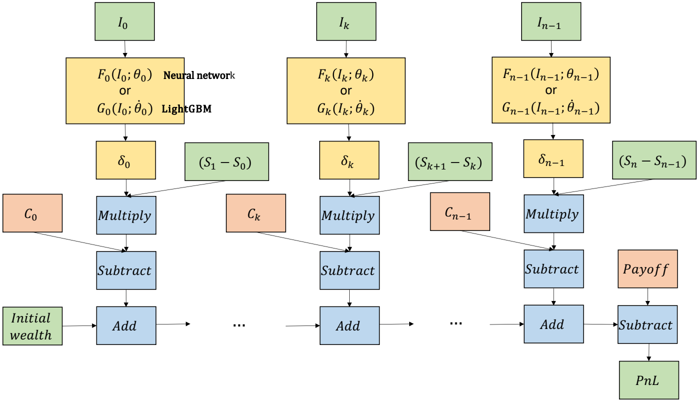
Methodology, X Hedging Framework



X Hedging: A novel hedging model

- Consisting of multiple LightGBM models.
- **Step 1:** fit all LightGBM models sequentially for one iteration.
- **Step 2:** train each model in random order to introduce stochasticity (inspired by SGD).

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X Hedging: A novel hedging model

- Consisting of multiple LightGBM models.
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- **Step 2:** train each model in random order to introduce stochasticity (inspired by SGD).
- Gradients and Hessians: use a computational graph conducting automatic differentiation.
- Smooth the regression lines by LightGBM models using a Savitzky–Golay filter.

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Loss functions

1. MSE (supervised learning)

$$\mathrm{MSE}(Y) = \frac{1}{N}\sum_{i=1}^{N}(\hat{Y}_i - Y_i)^2, \quad (1)$$

   where N is the number of samples, $\hat{Y}_i$ are the estimated target values, and Y_i are the true target values.

2. Quadratic CVaR (reinforcement learning)

$$\text{Quadratic CVaR}_\alpha = \frac{1}{|Y'|}\sum_{y' \in Y'}(y')^2. \quad (2)$$

Experiments

The performance of X Hedging is compared to Deep Hedging and the benchmark BS delta-hedging formulas (for cases with no market frictions and proportional transaction costs).

- Experiment 1: No market frictions and MSE as loss function.
- Experiment 2: Proportional transaction costs and Quadratic CVaR as loss function.

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Parameters

S_0	K	r	λ	σ	T	n	N
1	1	0.0	0.0	0.2	1	10	20000

Data leaf	Leaves	Boost rounds	Learn rate	Early stop	Iter
5	$10+j$	$10+5j$	0.1	10	15

Act. fun	Optimizer	Learn rate	Neurons	Batch size	Epochs
tanh	Adam	0.01	32	1024	10000

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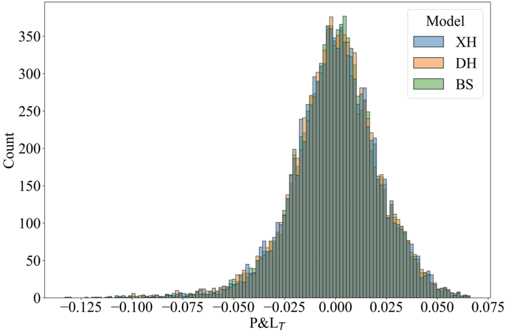
Experiment 1: No market frictions, MSE as loss function



Histograms for X Hedging (XH), Deep Hedging (DH), and BS Hedging (BS)

Experiment 1:No market frictions, MSE as loss function

	Mean	St.Dev	JS(DH‖z)	JS(XH‖z)	Time
BS	-0.000192	0.021327	0.000437	0.000398	10.0
DH	-0.000152	0.021415	-	0.000632	803.0
XH	-0.000146	0.021482	-	-	872.0

JS-divergence values: compute how close two probability distributions are.

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Experiment 1: No market frictions, MSE as loss function

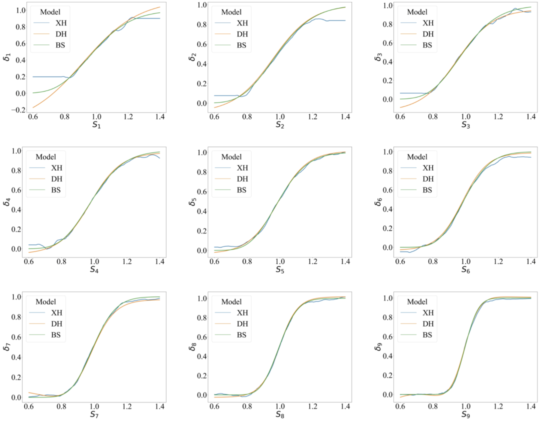

Figure: Hedging strategies

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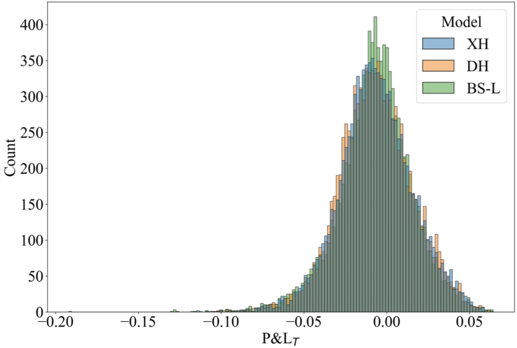
Experiment 2: proportional transaction costs and Quadratic CVaR as the loss function



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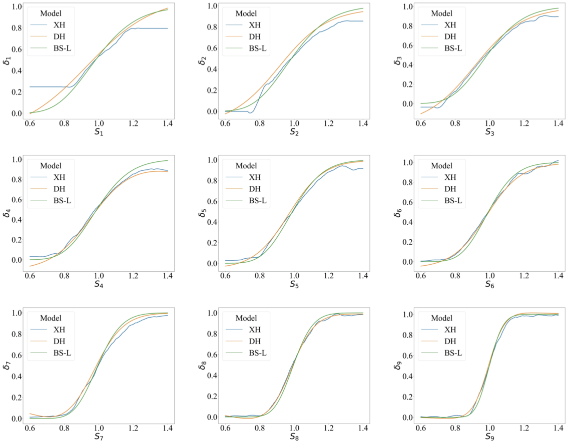
Experiment 2: Proportional transaction costs and Quadratic CVaR as the loss function

	Mean	Std	JS(DH‖z)	JS(XH‖z)	Time
BS-L	-0.006963	0.022263	0.00294	0.002170	10.0
DH	-0.006747	0.022440	-	0.001093	860.0
XH	-0.006626	0.022412	-	-	833.0

Experiment 2: Proportional transaction costs and Quadratic CVaR as the loss function



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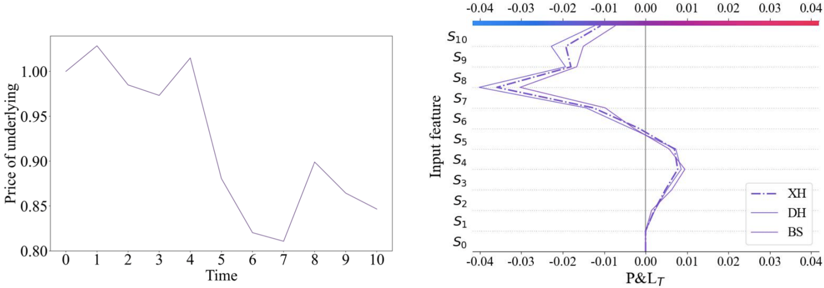
Summary: X Hedging v.s. Deep Hedging

- Both produce very similar results, and both are still satisfactory compared to benchmark analytical BS hedging.
- The hedging strategies are very similar to the later time steps. They are more dissimilar in the early time steps.
- In Experiment 2, all histograms are shifted to the left, indicating more losses (which is natural since a transaction cost is induced on every transaction).
- In conclusion, **X Hedging performs on par with Deep Hedging and benchmark BS hedging**.

SHAP decision plot for XH, DH and BS



(a) Underlying asset path movements for time index k.

(b) SHAP decision plot.

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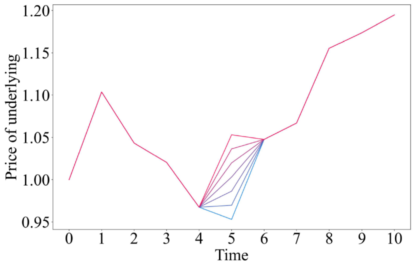
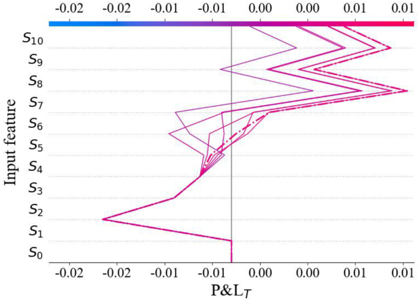
Sensitivity analysis when S_5 is changed and reverse after that



(a) Underlying asset path movements for time index k.



(b) SHAP decision plot.

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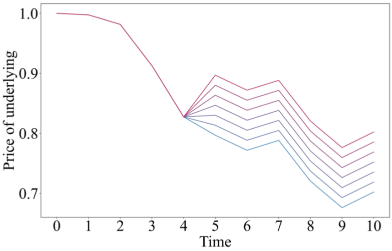
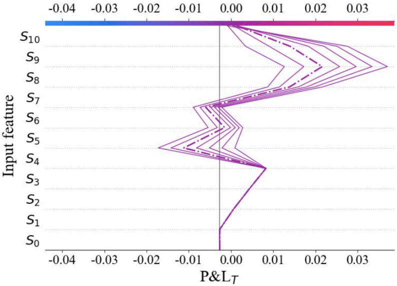
Sensitivity analysis on a small change in S_5 and parallel changes from there

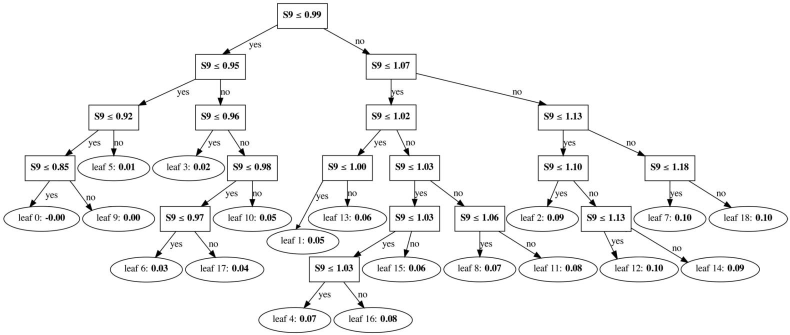


(a) Underlying asset paths movements for time index k.



(b) SHAP decision plot.

Local explainability: one decision tree visualised at $k = 9$



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Conclusion

- We proposed a novel option hedging framework, termed *X Hedging*.
- X Hedging is
  - a general framework that can handle different market frictions.
  - comparable to deep-hedging (using a neural network) in terms of performance.
  - inherently explainable.

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Thank you for your attention!

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