Identifying the number of latent factors of stochastic volatility models

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Motivation

- **Accurate specification of asset price dynamics** is of crucial importance in financial risk management, pricing and hedging of derivative securities.
- As is well known in financial mathematics, under the no-arbitrage condition, price processes must be **semimartingales**. A huge amount of work has been done in the last thirty years in order to render the assumptions about the data generating processes of the price series more in line with the empirical evidence.
- Traditionally, the construction of statistical tests and model calibration procedures requires the **use of option data**. However, the outcome of the tests may be rather sensitive to moneyness and maturity of the options.

Motivation

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- Conversely, the approach followed here simply requires the use of **prices of the underlying asset** and does not require any calibration procedures via option prices.
- The correct identification of an appropriate semimartingale model can nowadays benefit from the use of **high-frequency data**. Unfortunately, **market microstructure effects** make that more difficult.
- Our method is **fully non-parametric** and takes advantage from the well established ability of the **Fourier estimator [Malliavin and Mancino, 2002]** to estimate iterated co-variation processes from discrete (high-frequency) price observations.

Literature: specification tests for diffusion models

Methods based on low frequency observations: maximum likelihood estimator [Fermanian and Salanié, 2004]; minimization of weighted distances between non-parametric conditional densities [Altissimo and Mele, 2009]...

Methods based on infill asymptotics: [Corradi and White, 1999], [Dette and von Lieres und Wilkau, 2003], [Dette et al., 2006]; [Jacod and Podolskij, 2013] based upon a matrix perturbation method; [Kunimoto and Kurisu, 2021] based on the separating information maximum likelihood method; [Billio et al., 2012], [Figini et al., 2020], based on the principal components analysis and Granger-causality network.

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Procedures to test the assumption of a local volatility model for the price dynamics against the alternative of a stochastic volatility model: [Corradi and Distaso, 2007], [Podolskij and Rosenbaum, 2012], [Dette and Podolskij, 2008], [Zu, 2015], [Ait-Sahalia, 1996], [Zu and Boswijk, 2017], [Corradi and Swanson, 2011], [Corradi and Distaso, 2006].

The Identification Method

We consider a fairly general class of stochastic volatility models. Let $p(t)$ be the log-asset price observed at time t.

$$dp(t) = \alpha(t)dt + \sigma(t)dW_t^1, \tag{1}$$

$$d\sigma^2(t) = \beta(t)dt + \gamma(t)\, dW_t^2, \tag{2}$$

$$d\gamma^2(t) = \eta(t)dt + v_\gamma(t)\, dW_t^3, \tag{3}$$

where W^1, W^2, W^3 are (possibly) correlated Brownian motions on a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\in[0,T]}, P)$. The usual regularity conditions hold.

The Identification Method

The model's identification method is based on **determining the number of non-zero eigenvalues of the variance-covariance matrix** associated to the price process and the higher order covariances.
More precisely, denote by $\langle \, , \rangle$ the quadratic (co-)variation operation, and define the following volatilities:

$$\langle dp_t \, , \, dp_t \rangle / dt := A_t \, , \; \langle dA_t \, , \, dA_t \rangle / dt := B_t \, , \; \langle dB_t \, , \, dB_t \rangle / dt := C_t \, , \tag{4}$$

and cross-volatilities:

$$\langle dA_t \, , \, dp_t \rangle / dt := a_t \;\; , \; \langle dB_t \, , \, dp_t \rangle / dt := b_t \, , \; \langle dA_t \, , \, dB_t \rangle / dt := c_t \, . \tag{5}$$

The Identification Method

For every $t \in (0, 2\pi)$, consider the following the 3×3 Gram matrix

$$\Gamma_t = \begin{pmatrix} A_t & a_t & b_t \\ a_t & B_t & c_t \\ b_t & c_t & C_t \end{pmatrix} \tag{6}$$

and denote by $\lambda_1(t) \geq \lambda_2(t) \geq \lambda_3(t) \geq 0$ its eigenvalues.

The matrix $\Gamma(t)$ is the variance-covariance matrix for the SDEs system (1)-(2)-(3). Thus, it has **rank equal to one** if W^1, W^2, W^3 are perfectly correlated, as it is for the level dependent volatility models (like Black-Scholes or CEV model), **two** in the case of a stochastic volatility model (like Heston model), or **three** for the stochastic volatility of volatility models.

The Identification Method

More precisely, we will test the number of factors through the following three processes

$$\xi(t) := \frac{\lambda_1(t)}{\lambda_1(t) + \lambda_2(t) + \lambda_3(t)} \tag{7}$$

$$\theta(t) := \frac{\lambda_2(t)}{\lambda_1(t) + \lambda_2(t) + \lambda_3(t)} \tag{8}$$

$$\psi(t) := \frac{\lambda_3(t)}{\lambda_1(t) + \lambda_2(t) + \lambda_3(t)}. \tag{9}$$

In order to perform the model-free estimation of the eigenvalues, we need to estimate non-parametrically the entries of the matrix. To this aim, in the next section we exploit the **Fourier estimation method**

The Fourier estimator of iterated co-variations

Define, for $|k| \leq N$,

$$c_k(A_{n,N}) := \frac{2\pi}{2N+1} \sum_{|s| \leq N} c_s(dp_n) c_{k-s}(dp_n),$$

where

$$c_k(dp_n) = \frac{1}{2\pi} \sum_{j=0}^{k_n - 1} e^{-\mathrm{i} k t_{j,n}} \delta_j(p).$$

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Then $\widehat{A}_{n,N,N_A}(t) := \sum_{|k|<N_A} \left(1 - \frac{|k|}{N_A}\right) c_k(A_{n,N}) e^{\mathrm{i}kt}$.

The Fourier estimator of iterated co-variations

The knowledge of the Fourier coefficients of the latent instantaneous volatility $A(t)$ allows us to **handle this process as an observable variable and we can iterate the procedure** in order to estimate the other variance and co-variance functions.

The consistency and CLT of the estimators have been studied under suitable conditions on the cut-off frequencies $N = O(n)$, $N_A = O(n^{1/2})$, $M = O(n^{1/2})$, $M_B = O(n^{1/4})$, $L = O(n^{1/4})$, $L_C = O(n^{1/8})$, $M_a = O(n^{1/4})$, $L_b = O(n^{1/8})$ and $L_c = O(n^{1/8})$.

Uniform consistency of the eigenvalue estimation

Theorem

Under suitable assumptions on the cut-off frequencies, the following convergence in probability holds

$$\lim_{n,M,N,L,M_a,M_b,L_c\to\infty} \sup_{t\in(0,2\pi)} |\hat{\lambda}_j(t) - \lambda_j(t)| = 0, \tag{10}$$

for $j = 1, 2, 3$.

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Proof: By the Weyl's Perturbation Theorem, we get

$$\sup_{t\in(0,2\pi)} |\hat{\lambda}_j(t) - \lambda_j(t)| \leq \sqrt{3} \sup_{t\in(0,2\pi)} \|\hat{\Gamma}(t) - \Gamma(t)\|_\infty.$$

Simulation study and empirical analysis

- **Black-Scholes** and **Constant Elasticity of Variance** model
- **Heston** model
- **Stochastic Volatility of Volatility** model
- Intraday prices from the **S&P 500 index futures**, for the period from January 2, 2008 to December 31, 2008

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For each model, firstly, the entries of the Gram matrix are estimated by the **Fourier method**; Then, the eigenvalues $\hat{\lambda}_1(t)$, $\hat{\lambda}_2(t)$, $\hat{\lambda}_3(t)$ of the estimated matrix and the normalized quantities $\hat{\gamma}(t)$, $\hat{\beta}(t)$, $\hat{\rho}(t)$ are computed and examined.

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We use the **Euler-Maruyama discretization scheme** with a step-size equal to $\frac{2\pi}{21600}$ over the interval $[0, 2\pi]$.
Cut-off frequencies: $N = n/2$, $N_A = n^{1/2}$, $M = N_A/2$, $M_B = (16M)^{1/2}$, $L = M_B/2$, $L_C = (16L)^{1/2}$, $M_a = M_B$, and $L_b = L_c = L_C$.
The Fourier estimates of the eigenvalues are evaluated on $2L_C$ equally spaced points in the interval $(0, 2\pi)$.

The Black-Scholes model

$$dp(t) = \left(\alpha - \frac{1}{2}\sigma^2\right) dt + \sigma dW(t)$$

$$\Gamma_t = \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_1(t) = \sigma^2, \quad \lambda_2(t) = 0, \quad \lambda_2(t) = 0$$

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We expect that $\hat{\gamma}(t) \approx 1$, while $\hat{\beta}(t) \approx 0$ and $\hat{\rho}(t) \approx 0$.

The Black-Scholes model

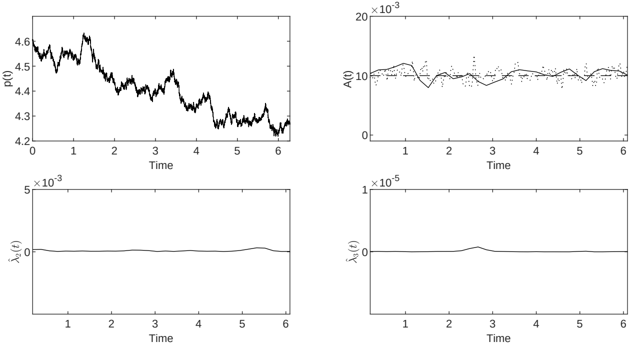

Figure: BS model. The first and the second panel show the log-price $p(t)$, the Fourier estimate of the spot volatility $A(t)$ (":"), the estimated eigenvalue $\hat{\lambda}_1(t)$ ("-") and the true spot volatility $A(t)$ ("- -") of the BS model. The third and the fourth panel plot the estimated eigenvalues $\hat{\lambda}_2(t)$ and $\hat{\lambda}_3(t)$.

The Black-Scholes model

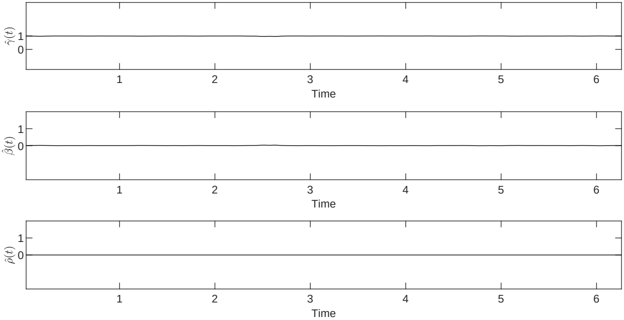

Figure: BS model. The figure shows the values of $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$ for the BS model.

The Black-Scholes model

Table: BS model. Some statistics on $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}_t$

	Min	Max	Mean	SD
$\hat{\gamma}(t)$	0.9672	1.0000	0.9928	0.0059
$\hat{\beta}(t)$	6.5236e-06	0.0328	0.0072	0.0059
$\hat{\rho}(t)$	0.0000	8.9366e-05	7.8658e-06	1.6236e-05

The Constant Elasticity of Variance model

$$dp(t) = \left(\alpha - \frac{1}{2}\sigma^2 \, e^{2p(t)(\delta-1)}\right) dt + \sigma e^{p(t)(\delta-1)} dW_t$$

$$\begin{aligned}
A(t) &:= \langle dp(t), dp(t)\rangle/dt = \sigma^2 e^{2p(t)(\delta-1)} && (11)\\
a(t) &:= \langle dA(t), dp(t)\rangle/dt = 2\sigma^4(\delta-1)e^{4p(t)(\delta-1)} && (12)\\
B(t) &:= \langle dA(t), dA(t)\rangle/dt = 4\sigma^6(\delta-1)^2 e^{6p(t)(\delta-1)} && (13)\\
b(t) &= \langle dB(t), dp(t)\rangle/dt = 24\sigma^8(\delta-1)^3 e^{8p(t)(\delta-1)} && (14)\\
C(t) &:= \langle dB(t), dB(t)\rangle/dt = 576\sigma^{14}(\delta-1)^6 e^{14p(t)(\delta-1)} && (15)\\
c(t) &:= \langle dA(t), dB(t)\rangle/dt = 48\sigma^{10}(\delta-1)^4 e^{10p(t)(\delta-1)} && (16)
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$$\lambda_1(t) = A(t) + B(t) + C(t), \qquad \lambda_2(t) = \lambda_3(t) = 0$$

The Constant Elasticity of Variance model

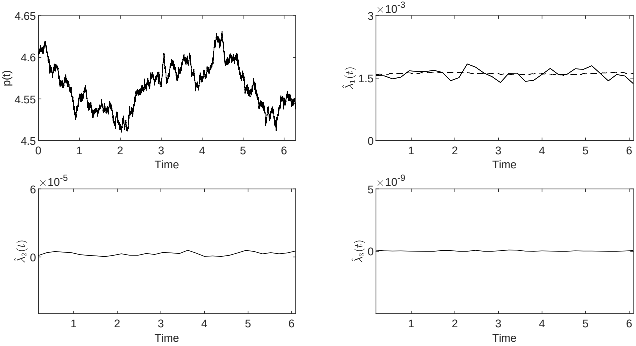

Figure: CEV model. The first panel shows the estimated eigenvalue $\hat{\lambda}_1(t)$ ("-") and the true eigenvalue $\lambda_1(t)$ ("- -")of the CEV model. The other panels plot the estimated eigenvalues $\hat{\lambda}_2(t)$ and $\hat{\lambda}_3(t)$.

The Constant Elasticity of Variance model

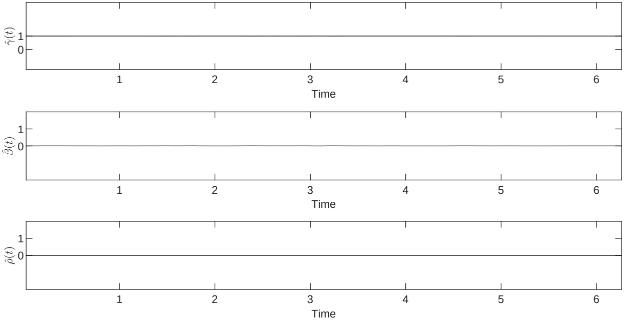

Figure: CEV model. The figure show the values of $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$ for the CEV model.

The Heston model

$$dp(t) = \left(\alpha - \frac{1}{2}\sigma^2(t)\right) dt + \sigma(t) dW_t^1 \tag{17}$$
$$d\sigma^2(t) = \kappa(\theta - \sigma^2(t))dt + \nu\sigma(t)dW_t^2, \tag{18}$$

$$A(t) = \sigma^2(t), \quad B(t) = \nu^2\sigma^2(t) \tag{19}$$
$$C(t) = \nu^6\sigma^2(t) \quad a(t) = \psi\nu\sigma^2(t), \tag{20}$$
$$b(t) = \psi\nu^3\sigma^2(t), \quad c(t) = \nu^4\sigma^2(t). \tag{21}$$

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\lambda_1(t) = \lambda_1\sigma^2(t), \quad \lambda_2(t) = \lambda_2\sigma^2(t), \quad \lambda_3(t) = 0,
$$

where

$$
\lambda_{1,2} = \frac{(1+\nu^2+\nu^6) \pm \sqrt{(1+\nu^2+\nu^6)^2 - 4(1-\psi^2)(\nu^2+\nu^6)}}{2}.
$$

The Heston model

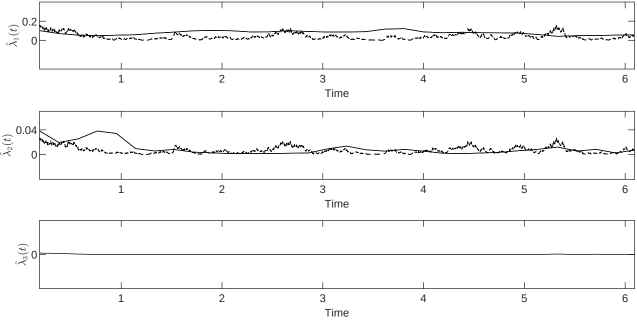

Figure: The first and the second panel show the estimated eigenvalues $\hat{\lambda}_1(t)$, $\hat{\lambda}_2(t)$ ("-") and the true eigenvalues $\lambda_1(t)$ and $\lambda_2(t)$ ("- -") of the Heston model. The last panel plots the estimated eigenvalue $\hat{\lambda}_3(t)$.

The Heston model

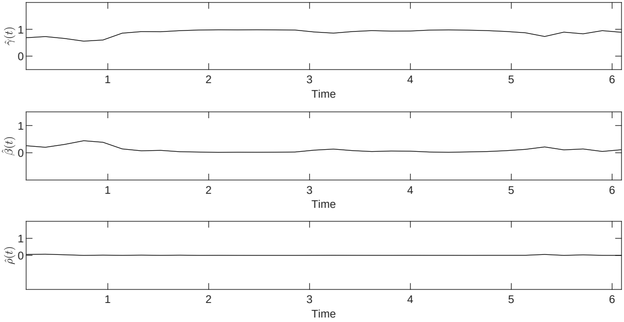

Figure: The figure shows the values of $\hat{\gamma}(t), \hat{\beta}(t)$ and $\hat{\rho}(t)$ for the Heston model.

The Heston model

Table: Some statistics on $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$

	Min	Max	Mean	SD
$\hat{\gamma}(t)$	0.5582	0.9814	0.8804	0.1178
$\hat{\beta}(t)$	0.0151	0.4418	0.1087	0.1082
$\hat{\rho}(t)$	0.0000	0.0697	0.0109	0.0180

The Stochastic Volatility of Volatility model

$$dp(t) = \left(\alpha - \frac{\sigma^2(t)}{2}\right) dt + \sigma(t) dW_t^1 \quad (22)$$

$$d\sigma^2(t) = \kappa(\theta - \sigma^2(t))dt + \xi(t) dW_t^2 \quad (23)$$

$$d\xi^2(t) = \kappa_\xi(\theta_\xi - \xi^2(t))dt + \nu_\xi \xi(t) dW_t^3, \quad (24)$$

$$A(t) = \sigma^2(t), \quad B(t) = \xi^2(t), \quad (25)$$

$$C(t) = \nu_\xi^2 \xi^2(t), \quad a(t) = \psi \xi(t)\sigma(t) \quad (26)$$

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$$\lambda_1(t) = \nu_\xi^2 \xi(t)^2$$

and

$$\lambda_{2,3}(t) = \xi(t)^2\sigma^2(t) \pm \sqrt{(\xi^2(t) + \sigma^2(t))^2 - \xi^2(t)\sigma^2(t)(1 - \psi^2)}.$$

The Stochastic Volatility of Volatility model

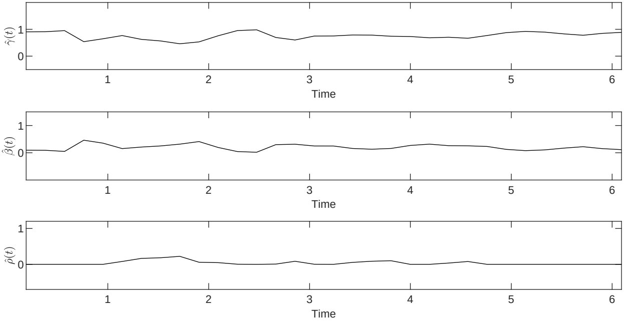

Figure: The figure shows the values of $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$ for the SVV model.

The Stochastic Volatility of Volatility model

Table: Some statistics on $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$

	Min	Max	Mean	SD
$\hat{\gamma}(t)$	0.4607	0.9809	0.7591	0.1346
$\hat{\beta}(t)$	0.0191	0.4608	0.2030	0.1076
$\hat{\rho}(t)$	0.0000	0.2230	0.0379	0.0602

Empirical study

We use the tick-by-tick data of the S&P 500 index futures for the period from January 2, 2008 to December 31, 2008. **Our dataset includes the collapse of Lehman Brothers**, the peak of the global financial crisis of 2007-2008.

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Cut-off frequencies: due to microstructure effects, the value for the cutting frequency N is derived using the procedure in [Mancino and Sanfelici, 2008]. The other frequencies are $N_A = (8n)^{1/2}$, $M = N_A/2$, $M_B = (16M)^{1/2}$, $L = M_B/2$, $L_C = (8L)^{1/2}$, $M_a = M_B$ and $L_b = L_c = L_C$.

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To keep these parameters large enough, we exclude days with less than 1000 trades from our sample, leaving us with 240 days.

Empirical study

Table: Some statistics on $\hat{\gamma}(t)$, $\hat{\beta}(t)$, $\hat{\rho}(t)$. $\hat{\lambda}_1(t)$, $\hat{\lambda}_2(t)$ and $\hat{\lambda}_3(t)$

	Min	Max	Mean	SD
$\hat{\gamma}(t)$	0.7171	1.0000	**0.9963**	0.0124
$\hat{\beta}(t)$	0.0000	**0.2819**	0.0037	0.0124
$\hat{\rho}(t)$	0.0000	**0.0123**	1.0412e-05	2.3767e-04
$\hat{\lambda}_1(t)$	1.5020e-06	0.0128	1.1871e-04	4.2712e-04
$\hat{\lambda}_2(t)$	1.8285e-06	0.0012	2.8133e-05	1.0558e-05
$\hat{\lambda}_3(t)$	0.0000	4.3946e-06	3.4117e-09	7.5914e-08

Empirical study

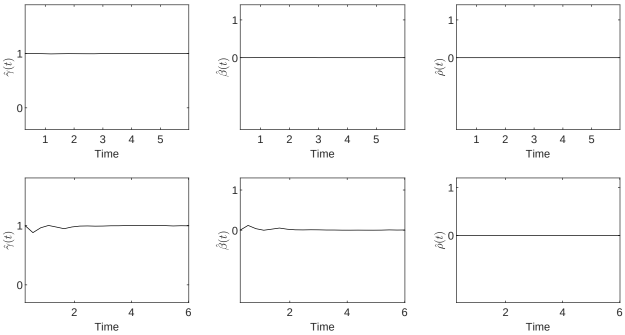

Figure: Panels 1-3 show the values of $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$ for the day **January 3, 2008**. Panels 4-6 show the values of $\hat{\gamma}(t)$, $\hat{\beta}(t)$ and $\hat{\rho}(t)$ for the day **October 10, 2008**.

Empirical study

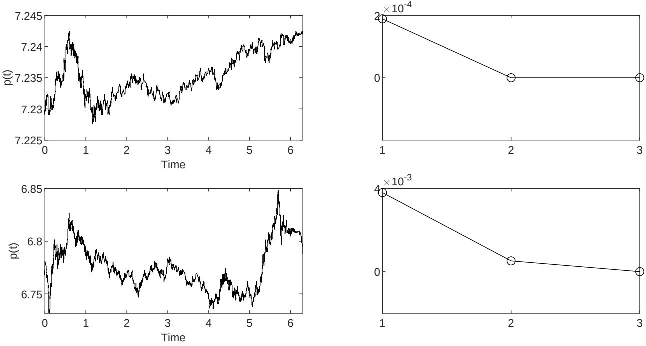

Figure: Panels 1 and 3 show the log-price $p(t)$ for the day January 3, 2008 and October, 10 2008. Panels 2 and 4 show the **scree plots** corresponding to the temporal instants of day January 3, 2008 and October, 10 2008 having, respectively, the maximum value of $\hat{\gamma}(t)$ and $\hat{\beta}(t)$.

Allaj, E., Mancino, M.E. and Sanfelici, S., (2023) *Identifying the number of latent factors of stochastic volatility models*. Working Paper.

https://sites.google.com/view/fourier-malliavinvolatilityest/home-page

THANK YOU

for your attention