Deep Quadratic Hedging

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ICCF24 – Amsterdam
April 2, 2024

Introduction: a financial problem

- Assume to be a financial institution who has sold a derivative contract and is confronted with a future liability: the payoff of the derivative at the terminal time;
- We want to find the optimal hedging strategy to replicates the contingent claim;
- For incomplete markets, we have a multitude of alternative approaches, among these, we find quadratic hedging approaches:
  - Mean-variance hedging, see [Bouleau and Lamberton (1989)], [Duffie and Richardson (1991)], [Schweizer (1994)];
  - Local risk minimization, see [Föllmer and Schweizer (1991)], [Schweizer (1991)], [Schweizer (1994)];
- In a Markovian diffusive setting, both mean-variance hedging and local risk minimization can be solved numerically by relying on PDEs, see [Heath, Platen and Schweizer (2001)].

Introduction: a numerical problem

- It is well known that numerical methods for PDEs suffer from the curse of dimensionality, hence it is problematic to apply quadratic hedging with a high number of risk factors.
- Our strategy relies on the following two observations:
  1. Both approaches can be treated from the point of view of BSDEs;
  2. High dimensional BSDEs can be efficiently solved by deep learning methods, see e.g. [E, Han and Jentzen (2017)], [Beck, E and Jentzen (2019)], [Huré, Pham and Warin (2020)], [Horvath, Teichmann and Žurič (2021)], [Barigou and Delong (2022)], etc.
- Our procedure consists then in:
  1. Expressing both approaches by means of the associated BSDEs;
  2. Applying the deep BSDE solver of [E, Han and Jentzen (2017)] to compute all the quantities of interest in a diffusive setting of arbitrary dimension.
- We show that deep learning-based methods extend the scope of applicability of quadratic hedging to higher dimensions.

Outline

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Hedging in incomplete markets
- The market model
- Local-risk minimization
- Mean-variance hedging
- The stochastic model

The Deep BSDE solver

Numerical experiments

The market model

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ a complete probability space, with $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ the **filtration generated by an** $(m+d)$**-dimensional Brownian motion** (W_t, B_t), with $W_t \in \mathbb{R}^m$ and $B_t \in \mathbb{R}^d$, $m \geq 1$ and $d \geq 0$.

We consider a financial market with:

- One **cash account**

$$\mathrm{d}S_t^0 = S_t^0 r_t \mathrm{d}t, \quad S_0^0 = 1; \tag{1}$$

- m **stocks**

$$\begin{cases} \mathrm{d}S_t^i = S_t^i \left(\mu_t^i \mathrm{d}t + \sum_{j=1}^m \sigma_t^{ij} \mathrm{d}W_t^j \right) \\ S_0^i = s_i \end{cases} \quad i = 1, \dots, m, \tag{2}$$

  where r, μ^i, σ^{ij} are $\mathbb{F}$-adapted processes such that existence and uniqueness for solutions is guaranteed.

Note: B is an additional Brownian motion entering into play, e.g., in the dynamics of r, μ or σ: whenever $d > 0$, the number of Brownian motions is larger than the number of risky assets and the market is incomplete.

Trading strategies

We consider a **trading strategy** $(\xi, \psi) \in \mathbb{R}^{m+1}$, where:

- $\xi_t := (\xi_t^1, \ldots, \xi_t^m)^\top$, with $\xi_t^i \in \mathbb{R}$ the **number of shares of the *i*-th stock** at time t,
- ψ_t the **units of cash account** at time t,

with associated (discounted) **value process**

$$\tilde{V}_t = \sum_{i=1}^{m} \xi_t^i \tilde{S}_t^i + \psi_t, \tag{3}$$

where $\tilde{S}_t^i := S_t^i / S_t^0$ is the **discounted stock prices** and $\tilde{S}_t^0 \equiv 1$.

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For a given initial wealth $\tilde{V}_0 = y$, the trading strategy (ξ, ψ) is **self-financing** if

$$\tilde{V}_t = y + \int_0^t \sum_{i=1}^{m} \xi_u^i \mathrm{d}\tilde{S}_u^i \tag{4}$$

(no inflows or outflows of cash).

The hedging problem

We want to **price and hedge** a European **contingent claim**:

- Let H a square-integrable $\mathcal{F}_T$-measurable random variable;
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Ideally, **the agent wants to reach the final condition**

$$\tilde{V}_T = H \quad \mathbb{P}\text{-a.s.} \tag{5}$$

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In our setting, **the market is incomplete**: for some claims, it is not possible to construct a self-financing strategy such that $V_T = H \, \mathbb{P}$-a.s. We must relax the structure of the set of strategies.

Approach 1: Local-risk minimization

- We insist on the fact that strategies should replicate the liability H, $\tilde{V}_T = H$ $\mathbb{P}$-a.s.;
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Proposition

The Föllmer-Schweizer decomposition of H is given by

$$H = X_0^{lr} + \int_0^T \eta_{1,s}^{lr,\top} \left(\mathrm{diag}(\tilde{S}_s)\sigma_s\right)^{-1} \mathrm{d}\tilde{S}_s + \int_0^T \eta_{2,s}^{lr,\top} \mathrm{d}B_s, \tag{6}$$

where $(X^{lr}, \eta_1^{lr}, \eta_2^{lr})$ *is the unique solution to the linear BSDE*

$$X_t = H - \int_t^T \eta_{1,s}^\top \mathrm{d}W_s - \int_t^T \eta_{2,s}^\top \mathrm{d}B_s - \int_t^T \eta_{1,s}^\top \phi_s \mathrm{d}s \tag{7}$$

with $\phi_t := \sigma_t^{-1}(\mu_t - r_t\mathbb{I})$ *the market price of risk.*

Approach 2: Mean-variance hedging

- We insist on the fact that strategies should be self-financing:
- **We accept a tracking error at time T.**

Following the approach of [Lim (2004)], the solution of this problem can be linked to the following system of two BSDEs:

$$\begin{cases} \mathrm{d}L_t = \left(|\phi_t|^2 L_t + 2\phi_t^\top \Lambda_{1,t} + \frac{\Lambda_{1,t}^\top \Lambda_{1,t}}{L_t} \right) \mathrm{d}t + \Lambda_{1,t}^\top \mathrm{d}W_t + \Lambda_{2,t}^\top \mathrm{d}B_t \\ L_T = 1, \; L_t > 0 \end{cases} \tag{8}$$

$$\begin{cases} \mathrm{d}X_t = \left(\phi_t^\top \eta_{1,t} - \frac{\Lambda_{2,t}^\top \eta_{2,t}}{L_t} \right) \mathrm{d}t + \eta_{1,t}^\top \mathrm{d}W_t + \eta_{2,t}^\top \mathrm{d}B_t \\ X_T = H \end{cases} \tag{9}$$

with $\phi_t = \sigma_t^{-1}(\mu_t - r_t \mathbb{I})$.

The model: a multidimensional Heston model

Let $m = d$, so that $W \in \mathbb{R}^d$ and $B \in \mathbb{R}^d$:

$$\begin{cases} \mathrm{d}\tilde{S}_t = \mathrm{diag}(\tilde{S}_t)\left(\left(A\,\mathrm{diag}(Y_t^2)\bar{\mu}\right)\mathrm{d}t + A\,\mathrm{diag}\left(Y_t\right)\mathrm{d}W_t\right), \\ \mathrm{d}Y_t^2 = \mathrm{diag}(\kappa)\left(\theta - Y_t^2\right)\mathrm{d}t + \mathrm{diag}(\sigma)\mathrm{diag}\left(Y_t\right)\left(\mathrm{diag}(\rho)\mathrm{d}W_t + \mathrm{diag}(\sqrt{\mathbb{I} - \rho^2})\mathrm{d}B_t\right), \end{cases} \tag{10}$$

where $\bar{\mu}, \kappa, \theta, \sigma, \rho \in \mathbb{R}^d$ and $A \in \mathbb{R}^{d \times d}$.

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- For $m = 1$ we retrieve a one-dimensional Heston model

$$\begin{cases} \mathrm{d}\tilde{S}_t = \tilde{S}_t\left(\mu Y_t^2 \mathrm{d}t + Y_t \mathrm{d}W_t\right), \\ \mathrm{d}Y_t^2 = \kappa\left(\theta - Y_t^2\right)\mathrm{d}t + \sigma Y_t\left(\rho \mathrm{d}W_t + \sqrt{1 - \rho^2}\mathrm{d}B_t\right), \end{cases} \tag{11}$$

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- We obtain a closed-form solution for the BSRE (8)

$$L_t = \exp\left\{\varphi(t, T) + \psi(t, T)^\top Y_t^2\right\},$$

  where φ and ψ satisfy a system of Riccati ODEs, generalizing the results in [Shen and Zeng (2015)] to the multidimensional case.
- We can then prove uniqueness by adapting the approach of [Shen and Zeng (2015)] to our setting, see Proposition 5.3.

Outline

The Forward-Backward SDEs

We can rewrite the problem like follows:

$$\mathcal{X}_t = x + \int_0^t b\,(s, \mathcal{X}_s)\,\mathrm{d}s + \int_0^t a\,(s, \mathcal{X}_s)^\top\,\mathrm{d}\mathcal{W}_s, \quad x \in \mathbb{R}^{2d} \tag{12}$$

$$\mathcal{Y}_t = \vartheta(\mathcal{X}_T) + \int_t^T h\,(s, \mathcal{X}_s, \mathcal{Y}_s, \mathcal{Z}_s)\,\mathrm{d}s - \int_t^T \mathcal{Z}_s{}^\top \mathrm{d}\mathcal{W}_s, \quad t \in [0, T], \tag{13}$$

where

- $\mathcal{W} = (W, B)^\top \in \mathbb{R}^{2d}$;
- The **forward process** is $\mathcal{X} = (\tilde{S}, Y^2)^\top \in \mathbb{R}^{2d}$;
- The **control process** is $\mathcal{Z} = (\mathcal{Z}^1, \mathcal{Z}^2)^\top$ with $\mathcal{Z}^1, \mathcal{Z}^2 \in \mathbb{R}^d$;
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The stochastic control problem

A solution $(\mathcal{Y}, \mathcal{Z})$ to (13) is a minimiser of

$$\min_{y=\mathcal{Y}_0,\ \mathcal{Z}=(\mathcal{Z}_t)_{t\in[0,T]}} \mathbb{E}\left[\left|\vartheta(\mathcal{X}_T) - \mathcal{Y}_T\right|^2\right]. \tag{14}$$

The Deep BSDE solver

The idea of the Deep BSDE solver of [E, Han and Jentzen (2017)] is to numerically solve a discretized version of (14). **Then, at each time step** n**, the control process** $\mathcal{Z}$ **is approximated by an artificial neural network (ANN).**

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- For $N \in \mathbb{N}$, we introduce a grid $0 = t_0 < t_1 < \ldots < t_N = T$ with step Δt s.t. $t_n = n\Delta t$;
- Let $\Delta \mathcal{W}_n = \mathcal{W}_{t_{n+1}} - \mathcal{W}_{t_n}$.

We consider an Euler-Maruyama discretization of (12)-(13), and introduce for each n, an ANN $\mathcal{N}_n : \mathbb{R}^{2d} \to \mathbb{R}^{2d}$. We get:

$$\mathcal{X}_{n+1} = \mathcal{X}_n + b(t_n, \mathcal{X}_n)\Delta t + a(t_n, \mathcal{X}_n)^\top \Delta \mathcal{W}_n, \qquad \mathcal{X}_0 = x, \tag{15}$$

$$\hat{\mathcal{Y}}_{n+1} = \hat{\mathcal{Y}}_n - h(t_n, \mathcal{X}_n, \hat{\mathcal{Y}}_n, \mathcal{N}_n)\Delta t + \mathcal{N}_n^\top \Delta \mathcal{W}_n, \qquad \hat{\mathcal{Y}}_0 = y. \tag{16}$$

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If now $\mathcal{P}((\mathcal{N}_n)_{n=1}^{N-1})$ denotes the set of parameters (i.e. weights and biases) of all the ANNs, then the stochastic control problem (14) becomes

$$\min_{y,\, \mathcal{P}((\mathcal{N}_n)_{n=1}^{N-1})} \mathbb{E}\left[\left|\vartheta(\mathcal{X}_N) - \hat{\mathcal{Y}}_N\right|^2\right]. \tag{17}$$

Outline

Benchmarks

- One-dimension: we compare the price process and the hedging strategies with the semi-explicit solutions
  - Local-risk minimization: [Heath et al. (2001)];
  - Mean-variance hedging: [Černý and Kallsen (2008)];
- Multi-dimension: we can only compare the price at time 0 by Monte Carlo for the process $\mathcal{X} = (\tilde{S}, Y^2)^\top$.

Local risk minimization

Portfolio dimension: 1	MC price: 6.854		
Time steps	**10**	**50**	**100**
BSDE solver price	6.829	6.846	6.855
Relative error (%)	**0.360**	**0.120**	**0.0162**
Training time (s)	128	735	1546
PDE price	6.850	6.850	6.850
Relative error (%)	**0.0488**	**0.0613**	**0.0618**
Portfolio dimension: 20	**MC price: 30.761**		
Time steps	**10**	**50**	**100**
BSDE solver price	30.704	30.783	30.828
Relative error (%)	**1.322**	**0.568**	**0.218**
Training time (s)	418	1993	3660
Portfolio dimension: 100	**MC price: 68.950**		
Time steps	**10**	**50**	**100**
BSDE solver price	68.269	68.427	69.020
Relative error (%)	**0.988**	**0.758**	**0.101**
Training time (s)	1772	9096	16527

Local risk minimization: $d = 1$, $N = 10$

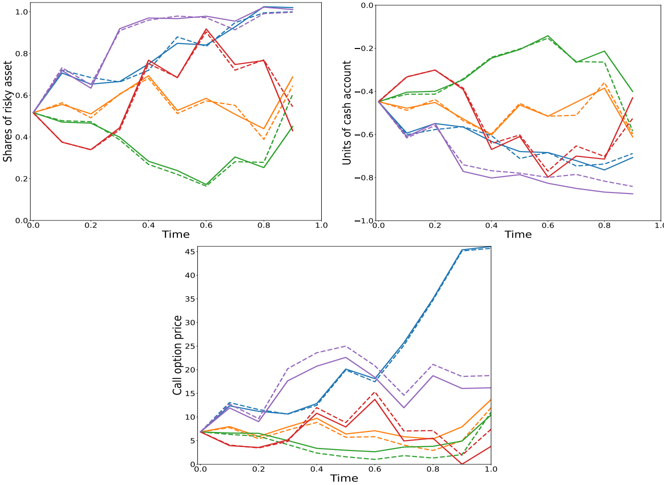

Figure: Deep solver solution (solid line) and benchmark solution (dashed line).

Local risk minimization: $d = 1$, $N = 50$

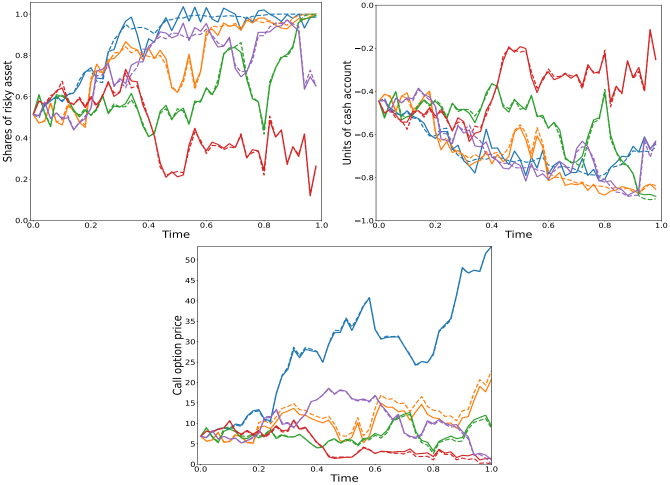

Figure: Deep solver solution (solid line) and benchmark solution (dashed line).

Local risk minimization: $d = 1, N = 100$

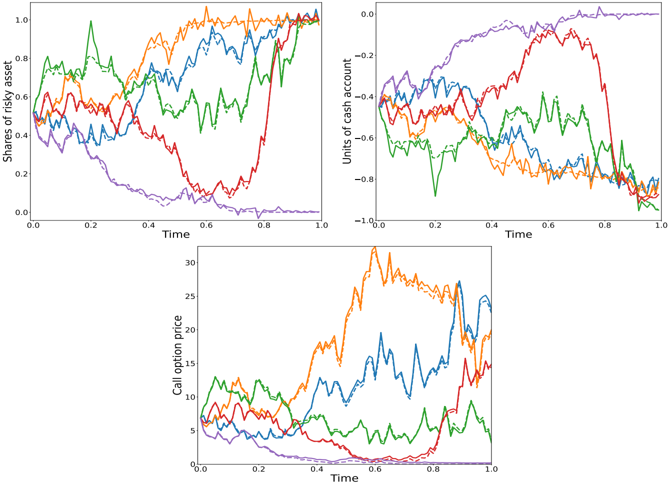

Figure: Deep solver solution (solid line) and benchmark solution (dashed line).

MSE local risk minimization: $d = 1$

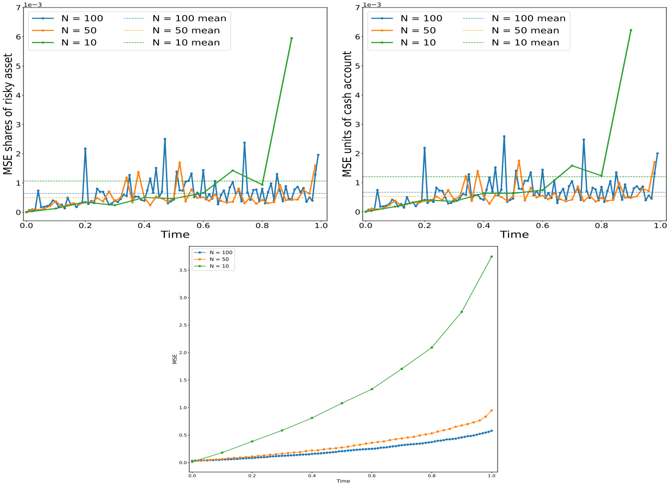

Figure: Above: shares of risky asset (left) and units of cash account (right); below: option price.

Mean-variance hedging

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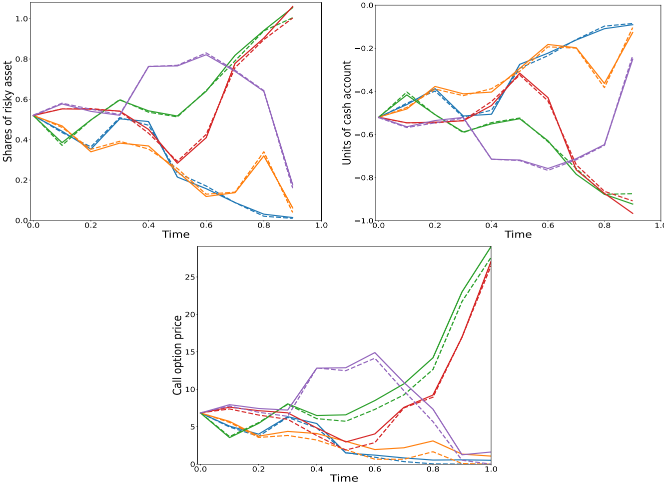
Portfolio dimension: 1		**MC price:**	**6.837**
		***L* value:**	**0.99984**
Time steps	**10**	**50**	**100**
BSDE solver *L* value	0.99969	0.99970	0.99969
Relative error (%)	0.01476	0.01434	0.01493
1st training time (s)	82	576	1048
BSDE solver price	**6.830**	**6.854**	**6.838**
Relative error (%)	**0.105**	**0.246**	**0.0250**
2nd training time (s)	1015	3270	5785
PDE price	6.853	6.853	6.853
Relative error (%)	0.245	0.233	0.232

Portfolio dimension: 100		**MC price:**	**68.831**
		***L* value:**	**0.97002**
Time steps	**10**	**50**	**100**
BSDE solver *L* value	0.97044	0.97007	0.97024
Relative error (%)	0.02936	0.00489	0.0230
1st training time (s)	1757	9860	20917
BSDE solver price	**68.168**	**68.892**	**68.910**
Relative error (%)	**0.964**	**0.0878**	**0.114**
2nd training time (s)	4516	21843	40253

Mean-variance hedging: $d = 1, N = 10$



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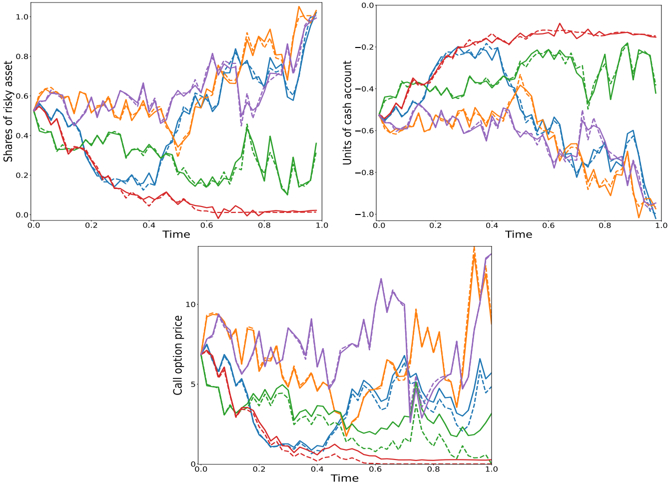
Mean-variance hedging: $d = 1$, $N = 50$



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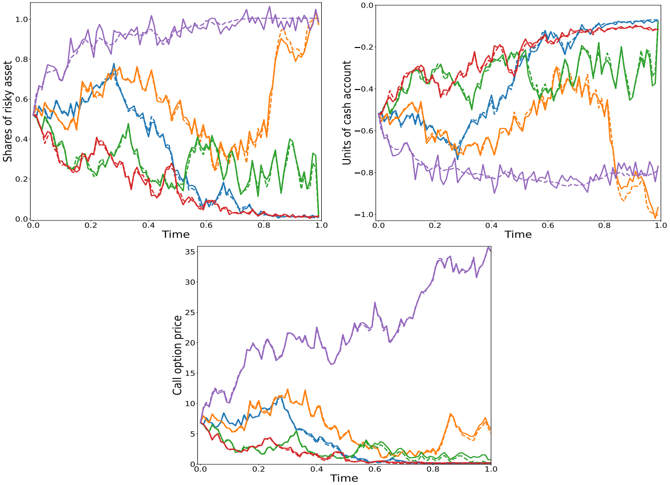
Mean-variance hedging: $d = 1, N = 100$



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MSE Mean-variance hedging: $d = 1$

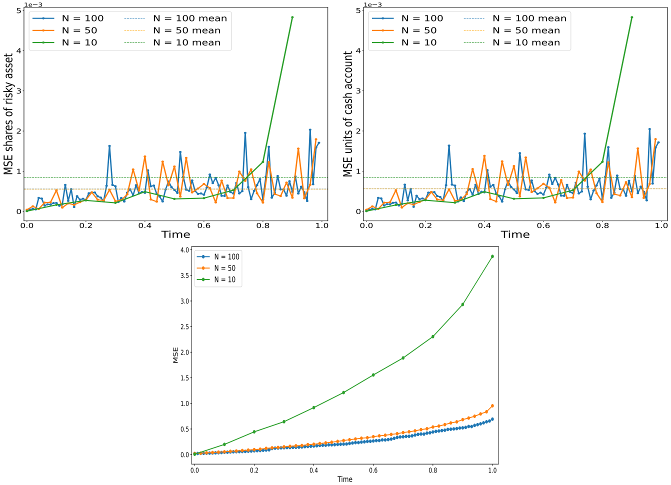

Figure: Above: shares of risky asset (left) and units of cash account (right); below: option price.

A note on the solver

- Initially, we assumed that the coefficients are $\mathbb{F}$-adapted, while [E, Han and Jentzen (2017)] works under the assumption of Markovianity: due to our modelling choice, it is natural for us to apply a Markovian solver;
- For non-Markovian model, such as the rough Heston model in [El Euch and Rosenbaum (2019)] and the rough-Bergomi model of [Bayer, Friz and Gatheral (2019)], the valuation equations take the form of BSPDEs which can be numerically solved by suitable extensions of the original solver of [E, Han and Jentzen (2017)], see e.g. [Bayer, Qiu and Yao (2022)], [Jacquier and Oumgari (2023)].
- The concrete mathematical structure of the model of choice will determine a certain variation of the reasoning we propose.
- Other deep learning-based solvers for BSDEs (or associated PDEs) in the Markovian setting can be found in the literature, see e.g. [Huré, Pham and Warin (2020)], [Beck et al. (2021)].
- We don’t exclude that other solvers could be also used in the same context.

References

- Lim, A. E. (2004). Quadratic hedging and mean-variance portfolio selection with random parameters in an incomplete market. *Mathematics of Operations Research*, 29(1), 132-161.
- Černý, A., and Kallsen, J. (2008). Mean–variance hedging and optimal investment in Heston's model with correlation. *Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics*, 18(3), 473-492.
- Delong, L. (2017). Backward Stochastic Differential Equations with Jumps and Their Actuarial and Financial Applications. *Springer*,
- Heath, D., Platen, E., and Schweizer, M. (2001). *Numerical comparison of local risk-minimisation and mean-variance hedging*. Option pricing, interest rates and risk management, 509-537.
- Schweizer, M. (2008). Local risk minimization for multidimensional assets and payment streams. *Banach Cent. Publ.*, 83, 213-229.
- E, W., Han, J., and Jentzen, A. (2017). Deep learning-based numerical methods for high-dimensional parabolic partial differential equations and backward stochastic differential equations. *Communications in Mathematics and Statistics*, 5(4), 349-380.
- Shen, Y. and Zeng, Y. (2015). Optimal investment–reinsurance strategy for mean–variance insurers with square-root factor process. *Insurance: Mathematics and Economics*, 62, 118-137.

References

- Lim, A. E. (2004). Quadratic hedging and mean-variance portfolio selection with random parameters in an incomplete market. *Mathematics of Operations Research*, 29(1), 132-161.
- Černý, A., and Kallsen, J. (2008). Mean–variance hedging and optimal investment in Heston's model with correlation. *Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics*, 18(3), 473-492.
- Delong, L. (2017). Backward Stochastic Differential Equations with Jumps and Their Actuarial and Financial Applications. *Springer*,
- Heath, D., Platen, E., and Schweizer, M. (2001). *Numerical comparison of local risk-minimisation and mean-variance hedging*. Option pricing, interest rates and risk management, 509-537.
- Schweizer, M. (2008). Local risk minimization for multidimensional assets and payment streams. *Banach Cent. Publ.*, 83, 213-229.
- E, W., Han, J., and Jentzen, A. (2017). Deep learning-based numerical methods for high-dimensional parabolic partial differential equations and backward stochastic differential equations. *Communications in Mathematics and Statistics*, 5(4), 349-380.
- Shen, Y. and Zeng, Y. (2015). Optimal investment–reinsurance strategy for mean–variance insurers with square-root factor process. *Insurance: Mathematics and Economics*, 62, 118-137.

Thanks for the attention!