Improved VaR/ES Backtests by using Self-Exciting Point Processes

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Agenda

1. Current state of VaR, ES, and their backtests
2. Principles of VaR and ES backtesting
3. Motivating example
4. What are Self-exciting Point Processes?
5. How to apply SEPP in VaR backtesting?
6. How to apply SEPP in ES backtesting?
7. Empirical example: the 2020 stock market crash
8. Wrap-up

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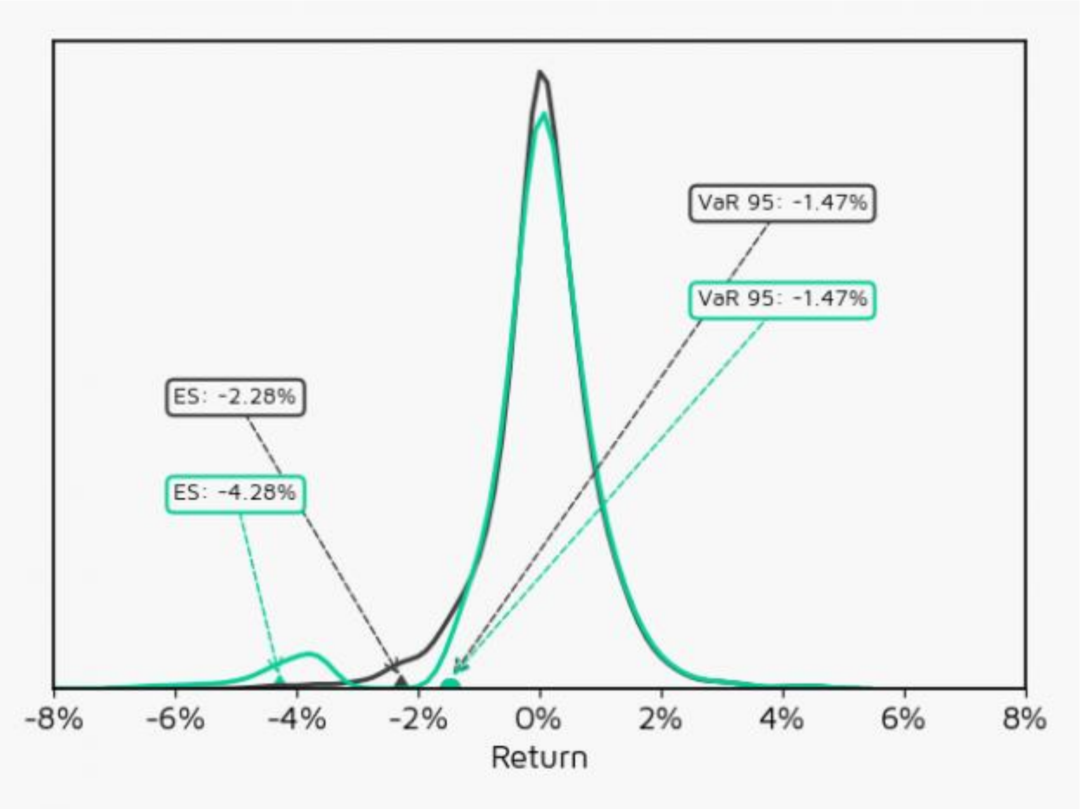
Current state of VaR, ES, and their backtests

- Capital requirements are broadly based on VaR
- VaR-based capital requirements suffer from a large deficiency: they do not capture tail risk
- Expected Shortfall (ES)*: the expected return in the worst α% of cases
- FRTB presents a move from VaR to ES due to VaR's inability to capture tail risk
- 97.5% ES used to determine FRTB IMA capital requirements
- Same approach used to calibrate FRTB SA
- Regulatory backtesting still on VaR

... or is it?



*ES is also known as CVaR, AVaR, expected tail loss and superquantile

Current state of VaR, ES, and their backtests

- 24 March 2023: EBA publishes draft rules for local supervisors
- Methodology for assessing internal models under FRTB
- Includes the requirement to conduct backtesting ES
- Banks see issues with:
  - Additional operational costs
  - No prescribed backtest
  - Lack of added value

Risk.net

EU banks balk at new market risk models back test

EBA proposals introduce additional expected shortfall back test for market capital risk models under FRTB

Current state of VaR, ES, and their backtests

Development of ES and VaR backtesting:

- VaR backtesting is a mature research area
- Unconditional (traffic light) and conditional (e.g., Christoffersen's independence test)
- ES backtesting is not so mature:
  - Took off in 2014
  - Next to a few unconditional backtests, only one (underperforming) conditional backtest

Literature studies and my research show that:

- Mainstream conditional VaR and ES backtests capture **time dependence** poorly
- VaR and ES backtests have low power in rejecting wrong risk models, especially when:
  - Sample size is low (< 500 observations)
  - High VaR levels are considered (> 97.5%)
- In other words: **when it matters**

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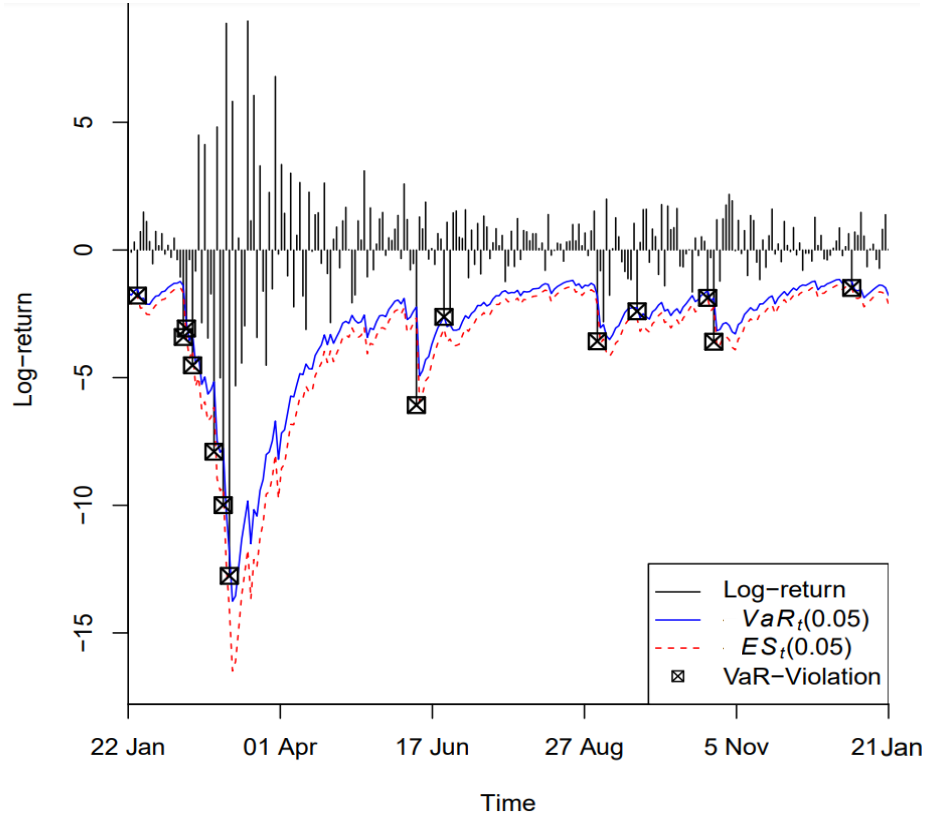
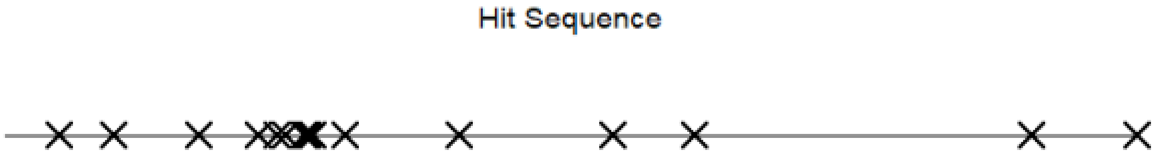
Principles of VaR and ES backtesting

VaR:





Statistical test: unconditional (traffic light) and conditional (e.g., Christoffersen's independence test)

ES: Take the mean returns at the hit times and perform simple t-test

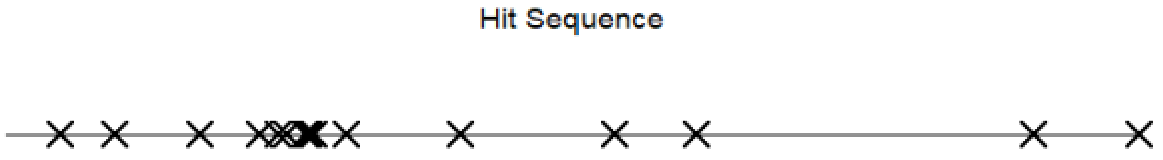
Principles of VaR and ES backtesting

Value-at-Risk:
Denote L_t as the trading desk's loss at time t and $\alpha \in (0,1)$, then, VaR equals:

$$\mathbb{P}(L_t > \mathrm{VaR}_\alpha^t | \mathcal{F}_t) = \alpha$$

Two main assumptions:

1. The unconditional assumption – the expected value of number of VaR outliers equals αT
2. The conditional assumption – at any time t, the probability of having a VaR outlier that day equals α (not α_t!)



Expected Shortfall:
Given cdf F_L of losses L_t, the ES is given by

$$ES_\alpha^t = \frac{1}{\alpha}\int_{1-\alpha}^{1} q_u(F_L)\, du$$

Leading to the following assumptions:

3. $\mathbb{E}[\mathrm{ES}_\alpha^t - \mathrm{L}_t \mid \mathrm{L}_t > \mathrm{VaR}_\alpha^t] = 0$
4. The VaR is correct

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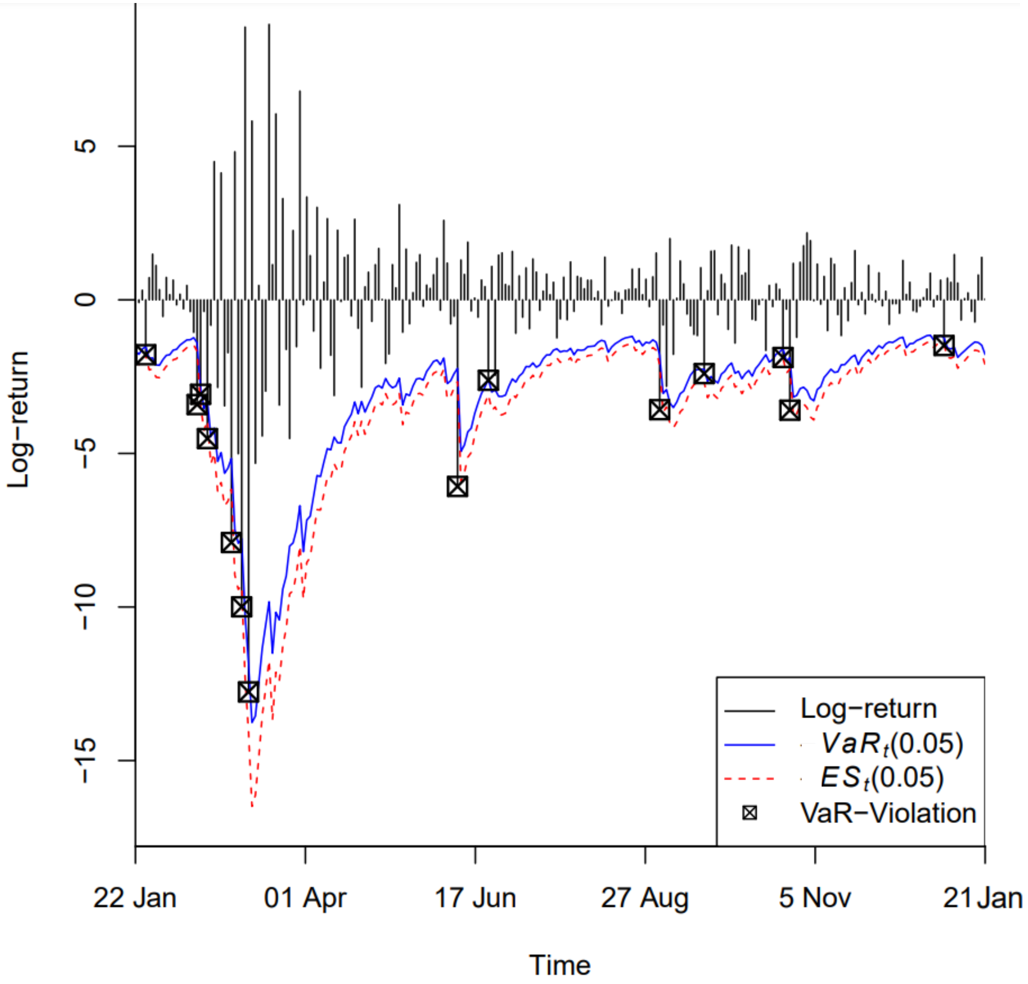
Motivating example

- S&P500 daily log-returns during the 2020 stock market crash
- 5%-VaR outliers: 14 (5.6%)
- Clear cluster of VaR outliers: 6 outliers in 15 days (40% >> 5%)
- Existing VaR and ES backtests (both unconditional and conditional) accept the risk model



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What are Self-exciting Point Processes?

- Originally developed for earthquake analysis
  - An event (earthquake/aftershock) excites more events soon after
  - The longer the wait, the lower the probability: back to steady state probability
- Involves modelling of the hit sequence (equals 1 if outlier, 0 if not)
- Arrival rate of a non-homogenous Poisson process modelled by

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{-\gamma(t-t_j)}$$

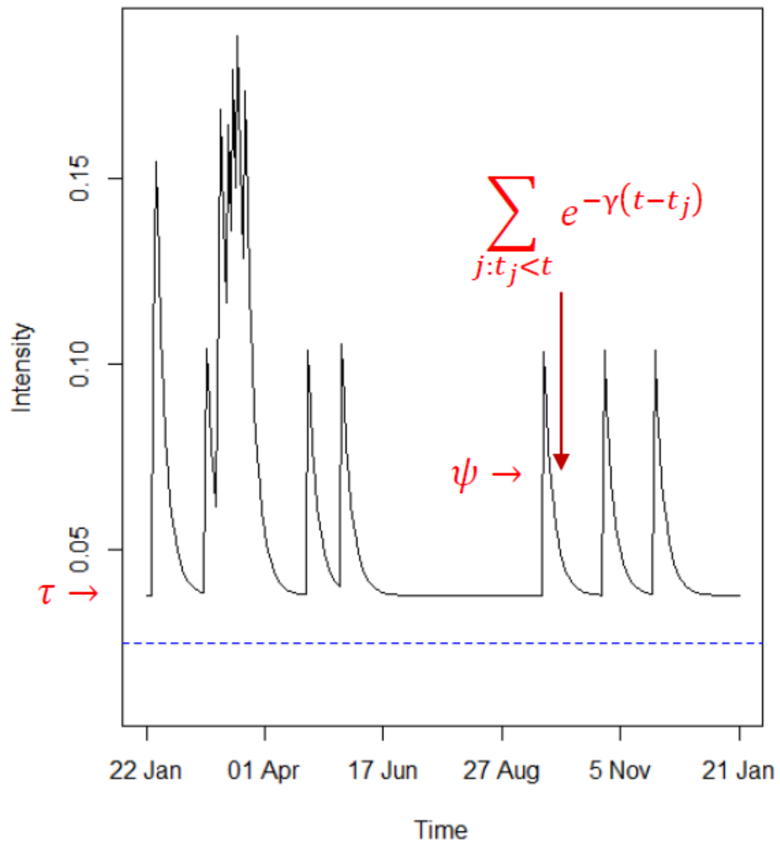
with $\tau, \gamma > 0$ and $\psi \geq 0$.

- **Many** applications: earthquakes, social media, crimes, deaths, etc.
- First financial application: Chavez-Demoulin (2005) in VaR modelling
  - SEPP's efficient modelling of tail behavior → Better VaR model

What are Self-exciting Point Processes?

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{-\gamma(t-t_j)}$$

- τ is the base intensity (arrival rate)
- ψ is the immediate jump in the intensity
- $\sum_{j:t_j<t} e^{-\gamma(t-t_j)}$ is the decay factor. As $t - t_j$, the distance between current time t and the time of the last violation t_j, gets larger, the intensity jump decays



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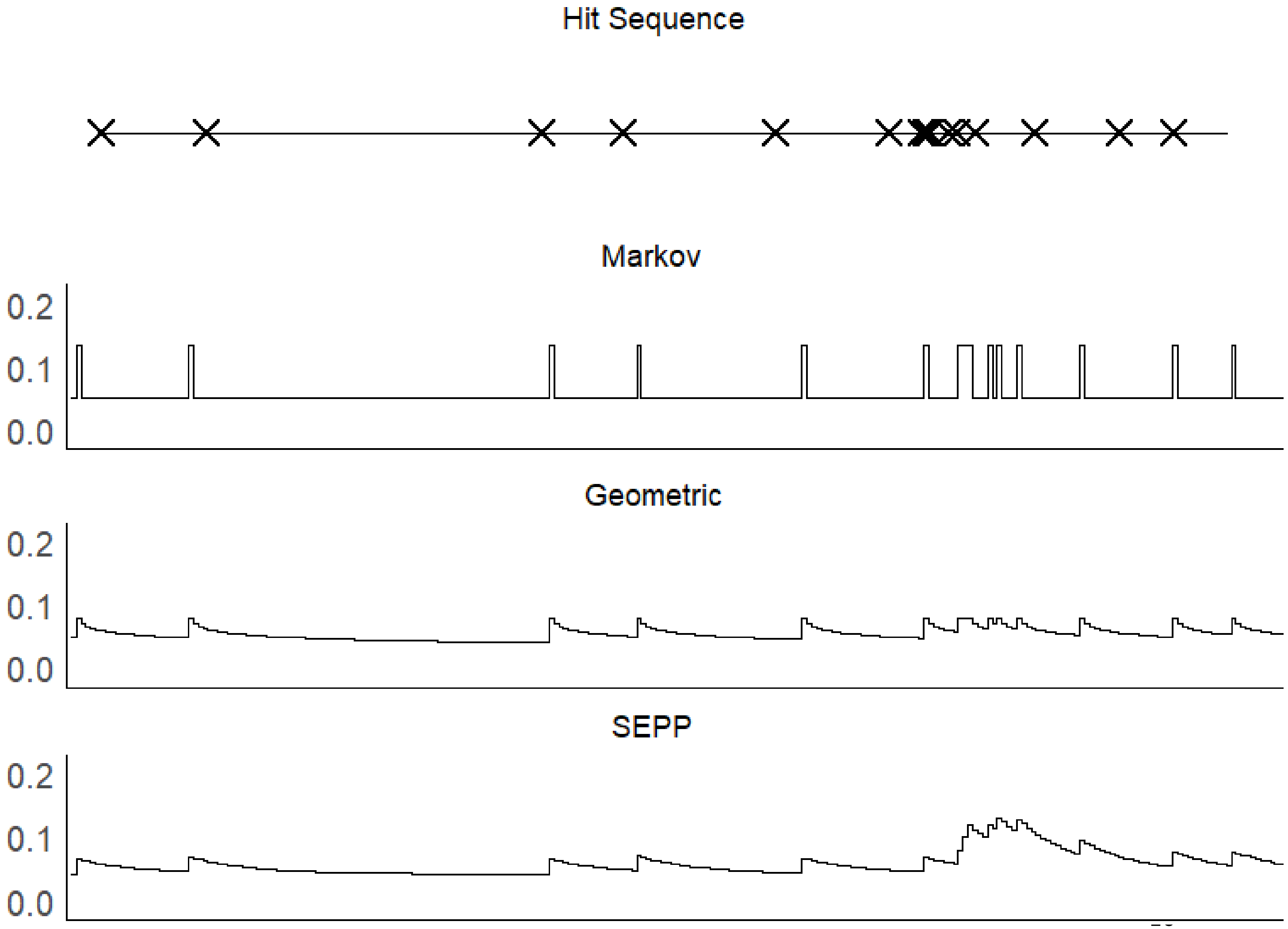
How to apply SEPP in VaR backtesting?

Crux lies in the modelling of the alternative hypothesis:

A better (?) alternative leads to higher power



Under the alternative:

- Christoffersen's (Markov) test does not fit clusters well
- Geometric test is better but limits itself by balancing between low and high intensity periods
- SEPP test is more flexible and better reflects intensity during the clustering period

How to apply SEPP in VaR backtesting?

- Standard simulation exercise in VaR backtesting, calibrated to four real business lines
- Across business lines, sample sizes, and VaR levels, SEPP backtests outperform most existing VaR backtests
- Christoffersen's (Markov) test has about half of the power of the SEPP-VaR test!

Sample size	Markov	Geometric	GV	CaViaR	SEPP	SEMPP	SEPP-VaR
Business line 1: $\sigma_u^2 = 6.344$, $\zeta = 0.913$							
250	0.258	0.310	0.401	0.469	0.380	0.406	**0.512**
500	0.284	0.416	0.623	0.561	0.487	0.557	**0.685**
750	0.319	0.537	0.801	0.670	0.583	0.678	**0.810**
1000	0.406	0.636	0.889	0.767	0.683	0.757	**0.890**
1250	0.410	0.737	**0.946**	0.840	0.771	0.828	0.944
1500	0.493	0.805	**0.972**	0.883	0.815	0.867	0.965
Business line 2: $\sigma_u^2 = 31.417$, $\zeta = 0.993$							
250	0.355	0.408	0.487	0.544	0.473	0.481	**0.597**
500	0.365	0.544	0.717	0.630	0.610	0.654	**0.763**
750	0.390	0.658	0.846	0.705	0.714	0.766	**0.860**
1000	0.433	0.747	0.910	0.760	0.794	0.831	**0.913**
1250	0.461	0.825	0.951	0.802	0.867	0.885	**0.956**
1500	0.530	0.883	**0.973**	0.855	0.908	0.925	0.971
Business line 3: $\sigma_u^2 = 2.768$, $\zeta = 0.923$							
250	0.107	0.068	0.101	**0.349**	0.111	0.120	0.261
500	0.079	0.052	0.200	**0.415**	0.090	0.119	0.410
750	0.074	0.056	0.409	0.531	0.085	0.123	**0.546**
1000	0.094	0.064	0.606	0.643	0.089	0.132	**0.659**
1250	0.094	0.080	0.747	0.724	0.106	0.142	**0.759**
1500	0.107	0.099	**0.841**	0.800	0.112	0.149	0.820
Business line 4: $\sigma_u^2 = 133.551$, $\zeta = 0.988$							
250	0.358	0.430	0.503	0.558	0.492	0.499	**0.622**
500	0.384	0.592	0.751	0.648	0.652	0.686	**0.799**
750	0.417	0.718	0.874	0.724	0.764	0.793	**0.883**
1000	0.471	0.805	0.932	0.781	0.845	0.861	**0.936**
1250	0.505	0.880	**0.968**	0.829	0.908	0.914	**0.968**
1500	0.579	0.929	**0.985**	0.882	0.942	0.941	0.982

Table 7: Power of conditional coverage tests for 97.5%-VaR in the four business lines from Berkowitz et al. (2011). The significance level is set at 10%. The highest power per scenario is marked in bold.

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How to apply SEPP in ES backtesting?

The importance of having (and not having) assumptions:

- To assess the bank's ES model, it is important to test **all four assumptions** underlying VaR and ES
- Current ES and joint VaR-ES backtests require the risk manager to **assume a return distribution**

  → Pandora's assumption box

  → Estimation error
- Sparked a move to assumption-free backtesting:
  - Only requires returns and reported VaR & ES
  - Two assumption-free unconditional backtests exist
- How can we obtain an assumption-free conditional backtest of VaR/ES?

How to apply SEPP in ES backtesting?

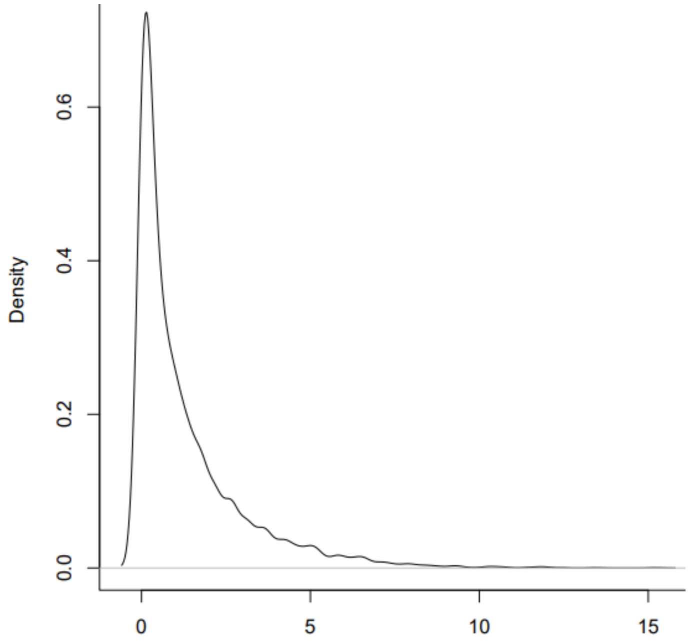
- Input: returns and VaR/ES predictions
- Adjust the intensity to include $A_{t_j} = L_{t_j} - ES_{t_j}$
- Compute the LR test statistic of the VaR test: $\mathcal{L}_1 \sim \chi^2_n$
- Perform t-test on ES and calculate a χ^2-distributed test statistic: $\mathcal{L}_2 \sim \chi^2_1$
- Under the assumption of conditional independence (EVT), we have:

$$\mathcal{L}_1 + \mathcal{L}_2 \equiv \mathcal{L}^* \sim \chi^2_{n+1}$$

n is complicated due to two non-trivial phenomenons:

- γ not identified under the null
- ψ is on the boundary of the parameter space under the null

→ Complicates the asymptotic distribution, but well-behaved



How to apply SEPP in ES backtesting?

Monte Carlo experiment (B = 1000) to compare existing 'assumption-free' backtests:

- H0: giving size, AR(1)-GARCH(1,1)-skewed student t distribution
- H1: Reported VaR 10% underestimated
- H2: Reported ES 10% underestimated
- H3: Reported VaR and ES 10% underestimated
- H4: Risk manager estimates normal innovations instead of the skewed student t
- H5: Risk manager estimates t-distribution instead of skewed student t distribution
- H6: estimating a GARCH(1,1)-skewed t, hence ignoring the AR(1) part

$\alpha = 5\%, n = 500$	SEPP ES	ESR1	ESR2	ESR3	ESR4	E-Backtest
H0	0.069	0.075	0.085	0.037	0.003	0.003
H1	**0.192**	0.074	0.084	0.037	0.003	0.006
H2	**0.163**	0.057	0.055	0.030	0.053	0.073
H3	**0.237**	0.057	0.055	0.030	0.053	0.076
H4	**0.920**	0.645	0.651	0.613	0.762	0.908
H5	**0.864**	0.242	0.231	0.218	0.386	0.556
H6	**0.115**	0.085	0.078	0.037	0.007	0.005

Preliminary results show outperformance across scenarios

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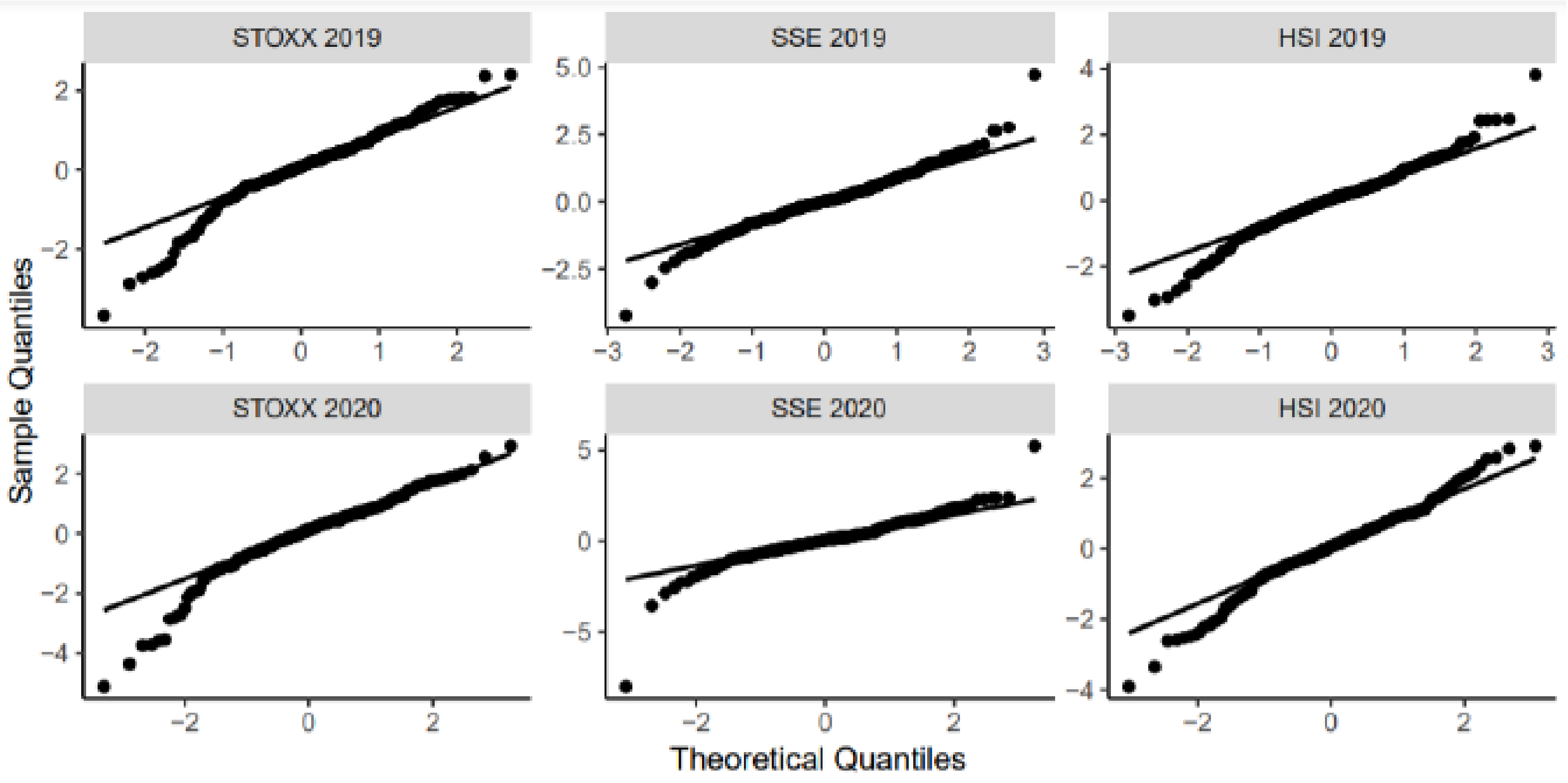
Empirical example: the 2020 stock market crash

- STOXX 600, Shanghai Composite, and Hang Seng
- AR(1)-GARCH(1,1) with Gaussian innovations is fitted
- Q-Q plots and normality tests reject Gaussian distribution of residuals



Empirical example: the 2020 stock market crash

- During 2020:

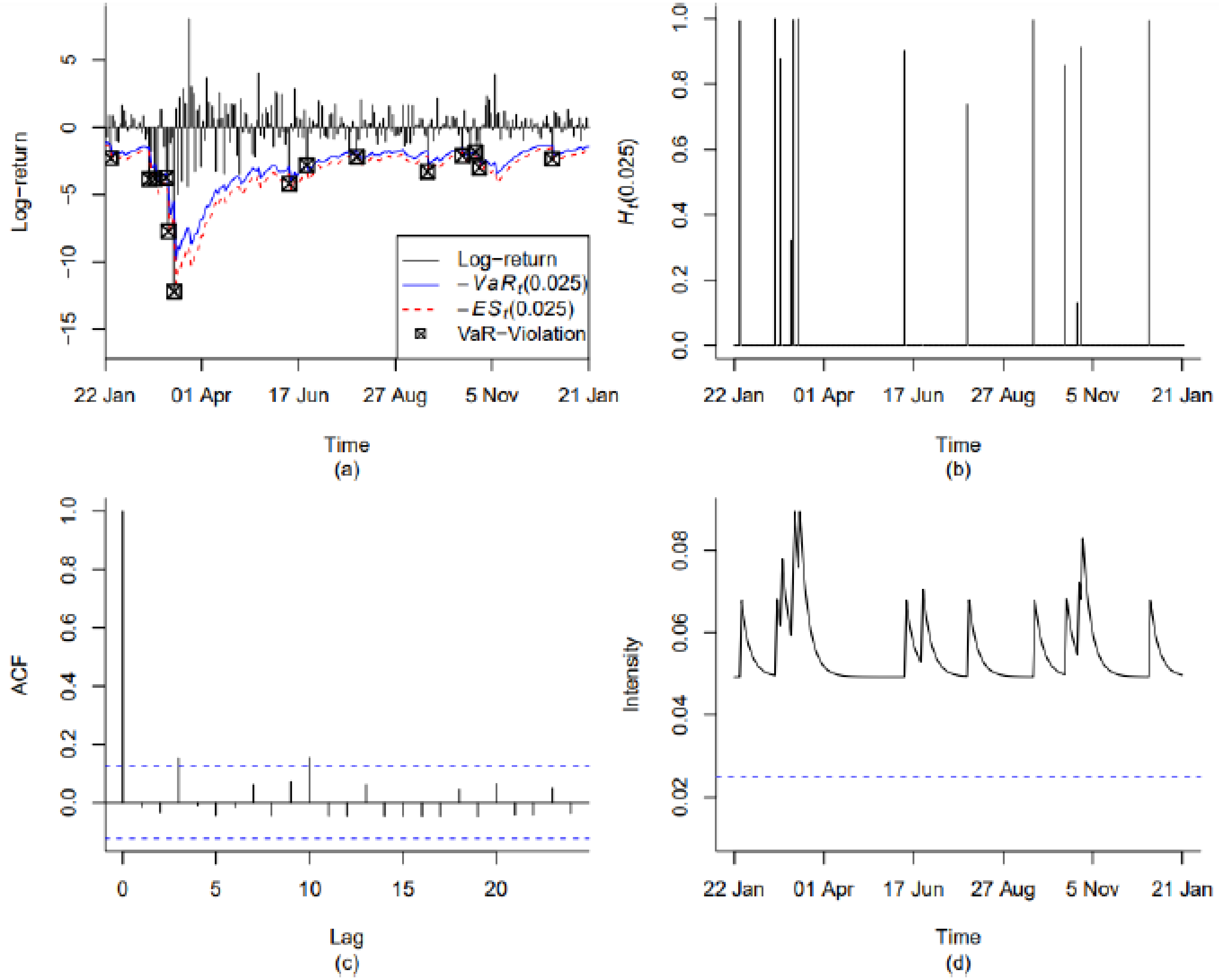


Figure 5-2. STOXX: VaR & ES (a), cumulative violations (b), ACF of cumulative violations (c), and estimated intensity process (d)

Empirical example: the 2020 stock market crash

- During 2019:

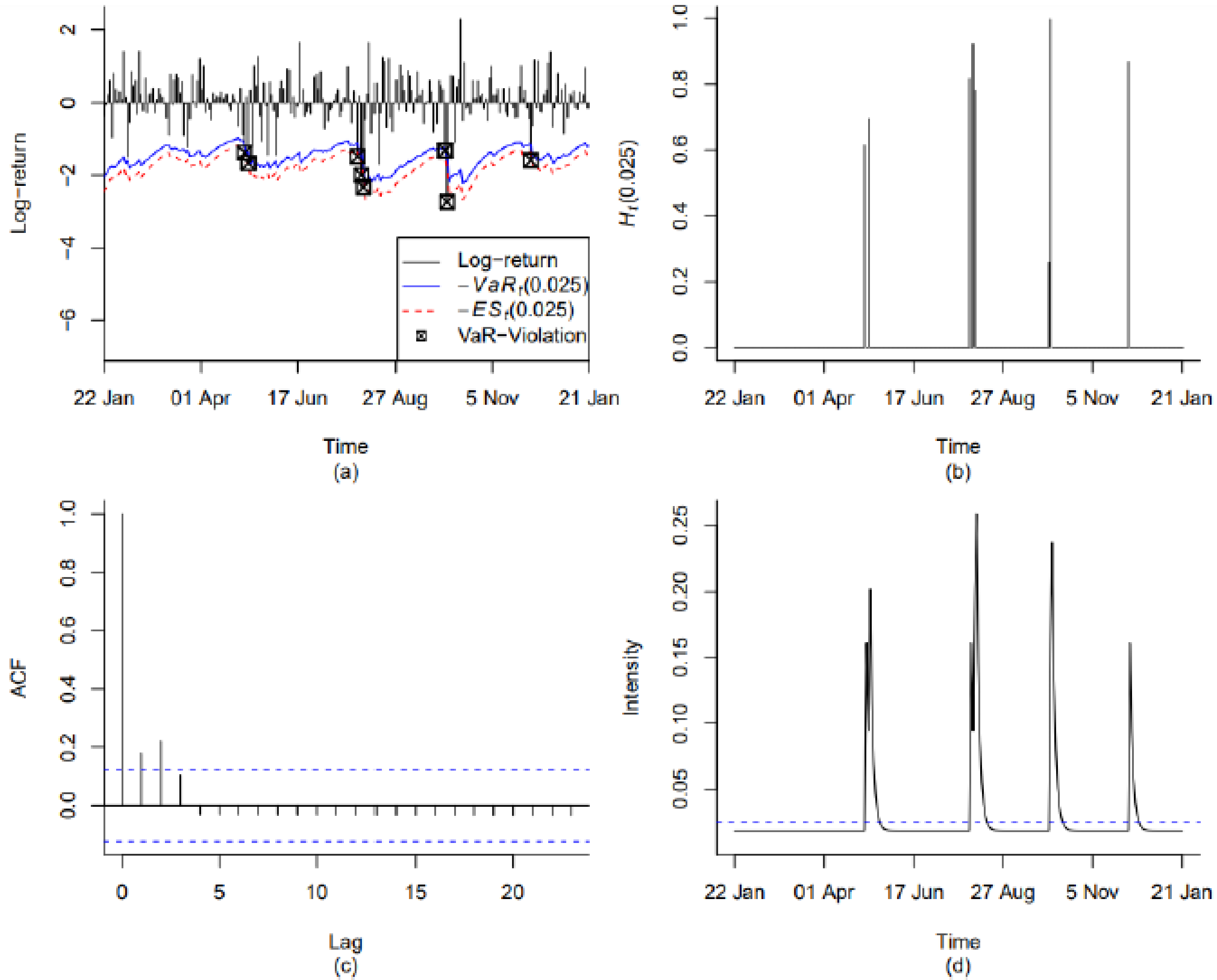


Figure 5-5. STOXX: VaR & ES (a), cumulative violations (b), ACF of cumulative violations (c), and estimated intensity process (d)

Empirical example: the 2020 stock market crash

- During 2020:

Table 5-4. Backtesting p-values

α	Index	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
0.025	STOXX	**0.003**	**0.006**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.014**	**0.021**	0.149	0.172
	SSE	0.112	**0.004**	0.121	**0.000**	0.218	**0.000**	**0.000**	0.370	0.959	**0.021**	**0.035**
	HSI	0.187	0.271	**0.002**	**0.002**	**0.027**	**0.001**	**0.002**	**0.033**	0.696	**0.000**	0.351
0.05	STOXX	**0.000**	**0.000**	**0.004**	**0.000**	**0.000**	**0.000**	**0.000**	**0.030**	**0.034**	0.223	0.324
	SSE	0.058	**0.000**	0.558	**0.004**	0.218	**0.000**	**0.000**	0.718	0.978	0.278	0.281
	HSI	**0.006**	**0.019**	**0.050**	**0.003**	**0.027**	**0.000**	**0.000**	0.056	0.564	**0.001**	0.611

- During 2019:

Table 5-6. Backtesting p-values

α	Index	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
0.025	STOXX	**0.034**	0.064	0.254	**0.028**	0.066	**0.002**	**0.002**	0.190	0.760	**0.000**	0.924
	SSE	0.182	**0.017**	0.934	0.136	0.684	**0.003**	**0.002**	0.844	0.992	0.996	1.000
	HSI	0.072	0.058	0.240	**0.028**	0.063	**0.032**	**0.021**	0.200	0.964	0.259	0.705
0.05	STOXX	0.157	0.085	0.400	0.096	0.066	**0.026**	**0.019**	0.445	0.891	**0.000**	0.016
	SSE	0.106	**0.005**	0.462	0.428	0.684	**0.002**	**0.002**	0.737	0.990	0.894	0.907
	HSI	0.059	**0.039**	0.333	**0.044**	0.063	**0.043**	**0.030**	0.265	0.977	0.552	0.614

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Wrap-up

- SEPP implementation in VaR and ES backtesting significantly boosts power by providing a more flexible outlier intensity specification
- SEPP ES backtest is the first powerful conditional ES backtest and more powerful than unconditional ES backtests
- Only requires PnL and reported VaR & ES
- Looking ahead:
  - ES backtesting will become relevant to banks with the introduction of FRTB (especially when EBA pushes through its RTS)
  - With a lack of powerful ES backtests and no guidance from the regulator, this will become a difficult task
  - A move towards joint VaR-ES backtesting is more attractive (you have the information anyway!)
  - The SEPP ES backtest is a powerful solution
- If you are interested on working on this topic, please reach out!

Thank you!