Dr. Pascal Halffmann, Yannick Becker – Fraunhofer ITWM

Risk Management in Portfolio Optimization: A Multicriteria Approach

ICCF 24 – Workshop Computational Finance

Recap: The traditional Framework of Markowitz‘ Portfolio Theory

Markowitz‘ Portfolio Theory

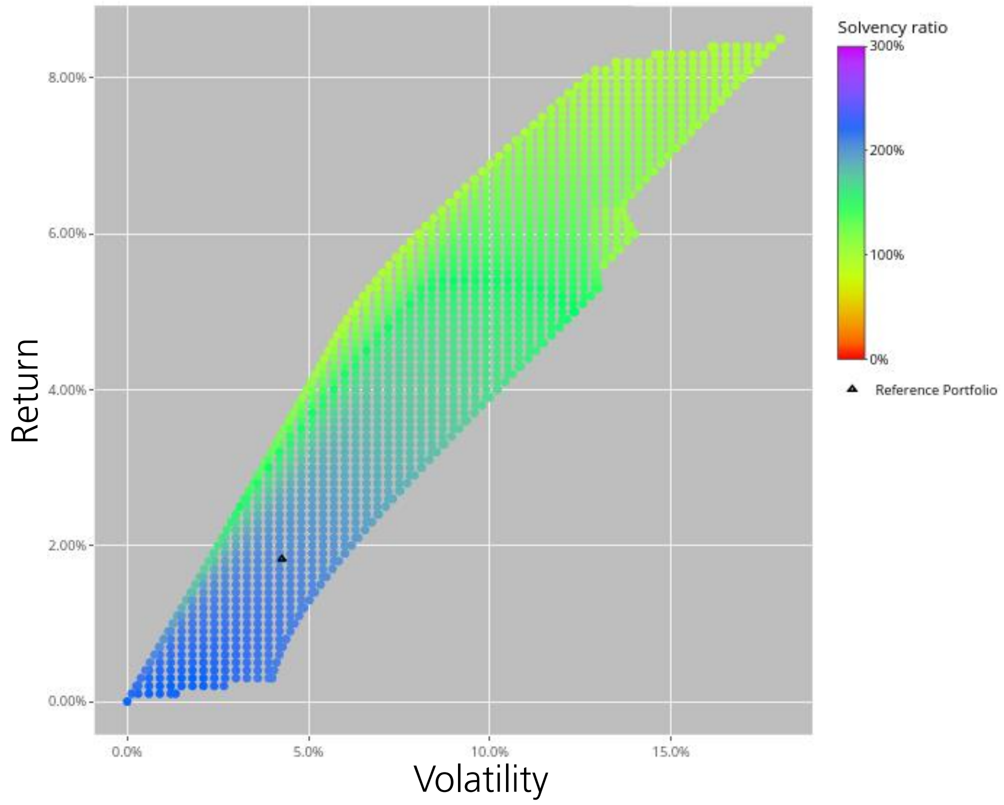
Mean-risk Model:

$$max_{w \in [0,1]^n} \mu^T w - q \cdot w^T \Sigma w,$$

where

- μ expected return vector
- Σ covariance matrix
- q „risk appetite“
- w vector of asset share such that $\sum_{i=1}^{n} w_i = 1$.

Harry M. Markowitz: Portfolio Selection. In: Journal of Finance. 7, 1952, ISSN 0022-1082, S. 77–91.



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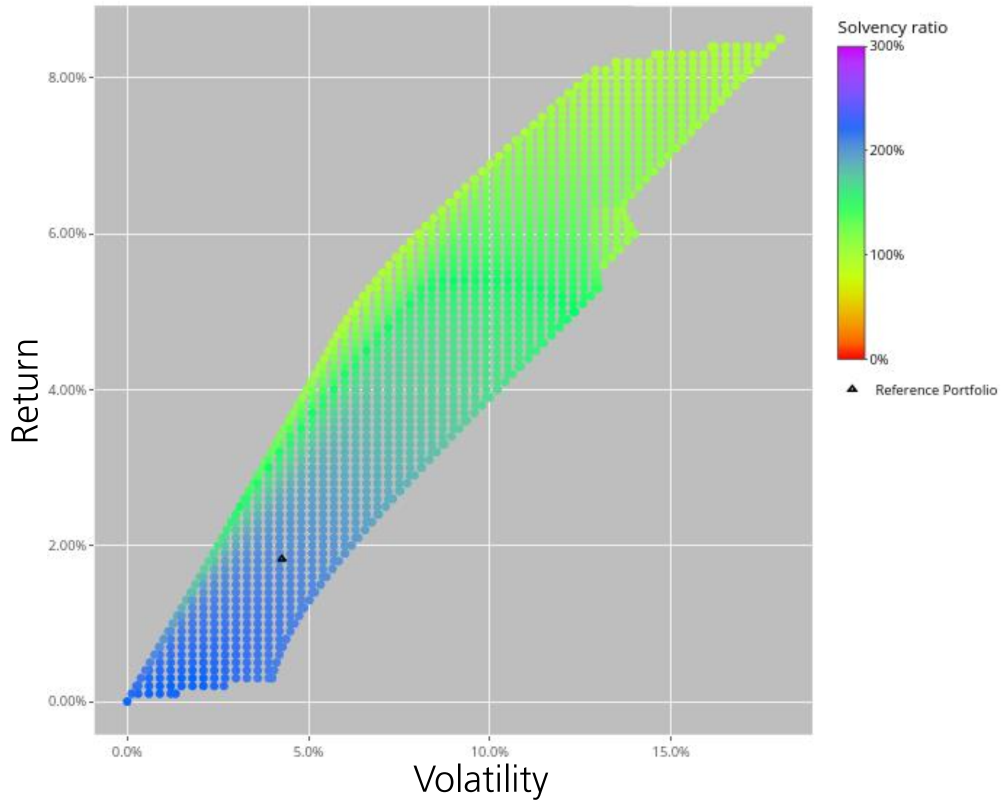
Limitations:

- **Only one solution**
- **Predetermined risk appetite**
- **Other objectives not incorporated**



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Recap: Multicriteria Portfolio Optimization

Extension of Markowitz‘ Portfolio Theory

Multicriteria Mean-risk Model:

$$\max_{w} \quad \mu^T w$$
$$\min_{w} \quad w^T \Sigma w$$
$$s.t. \quad w \in [0,1]^n, \quad \sum_{i=1}^{n} w_i = 1$$

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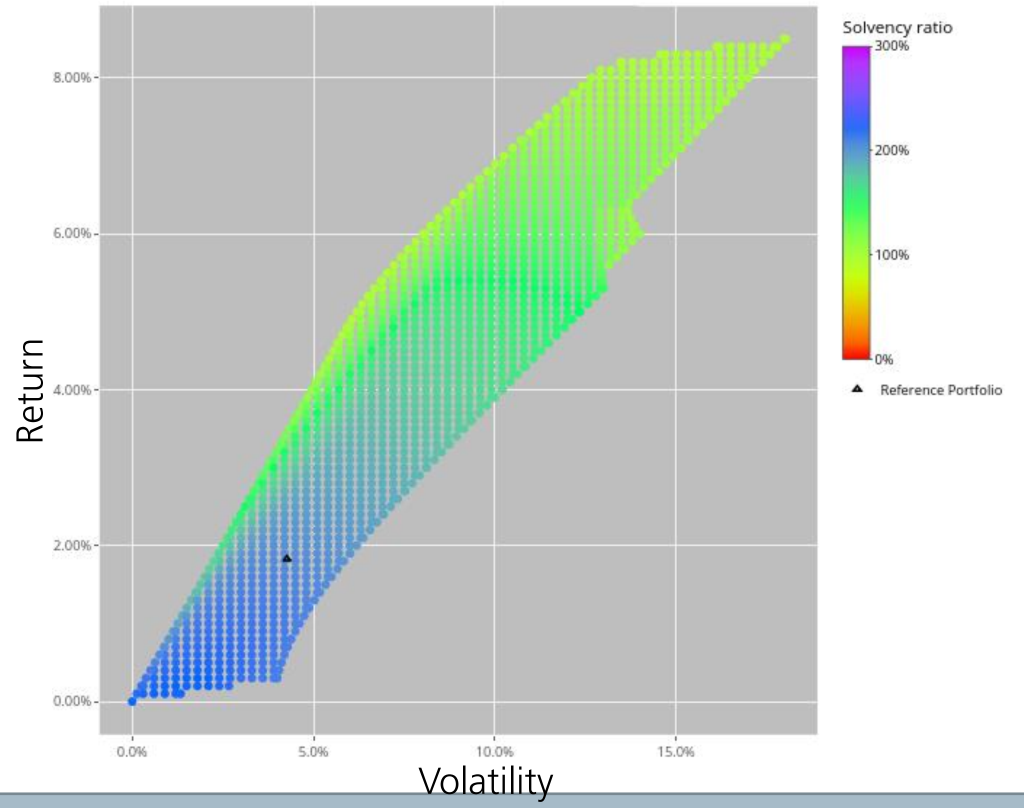

Sawik, B.: "Survey of multi-objective portfolio optimization by linear and mixed integer programming", Applications of Management Science, Vol. 16, pp. 55-79 (2013).
https://doi.org/10.1108/S0276-8976(2013)0000016007

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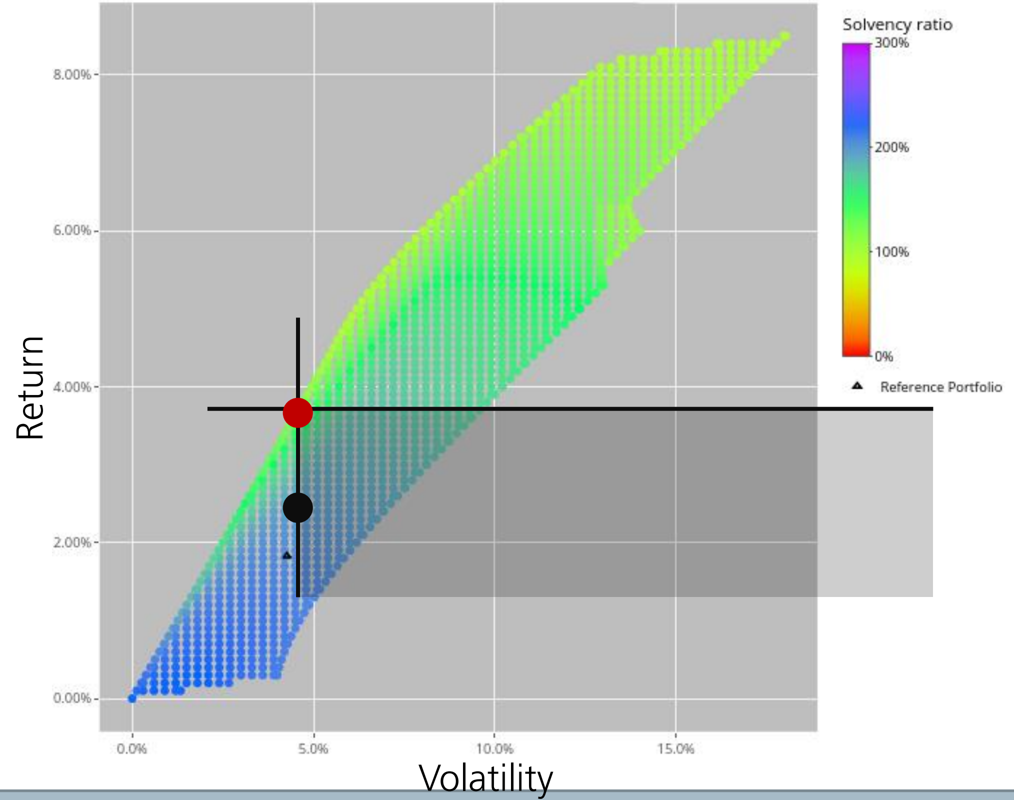

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Limitations:

- **Estimated parameters μ and Σ are uncertain**

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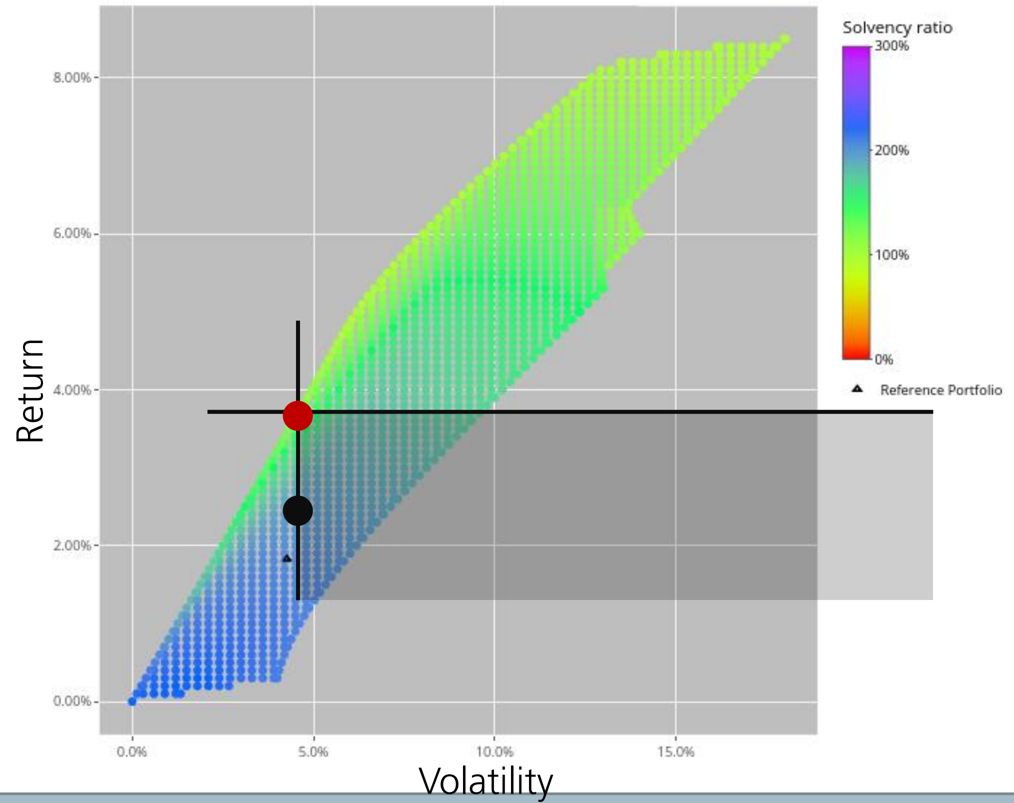

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Multicriteria optimization is a valuable tool to model and solve real-world problems. A prime example is portfolio optimization.

Can we use multicriteria optimization as a decision support tool for coping with uncertain parameters?«

Sneak Peek: Multicriteria-Based Sensitivity Analysis of Return Parameters

Problem Outline

1. Expected return of assets has to be computed a priori
2. Computation based on historical, empirical data, return parameter is a random variable itself
3. Different ways to compute, leads to discussion between broker

Has a variation in a return parameter an impact on the outcome of the asset allocation?

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Idea (focus on one reference portfolio and one asset):

Given μ_{min} and μ_{max} for the first asset, rewrite $f_{return}(w) = \mu^T w$:

$$f_{return}(w,\lambda) = \sum_{i=2}^{n} w_i \mu_i + w_1(\lambda \mu_{min} + (1-\lambda)\mu_{max}),$$

with $\lambda \epsilon [0,1]$. For $W = \{w \in [0,1]^n : \sum w_i = 1\}$, solve

$$\min_{w \in W} f(w,\lambda) = \begin{pmatrix} f_{return}(w,\lambda) \\ f_{volatility}(w) \\ \vdots \end{pmatrix}.$$

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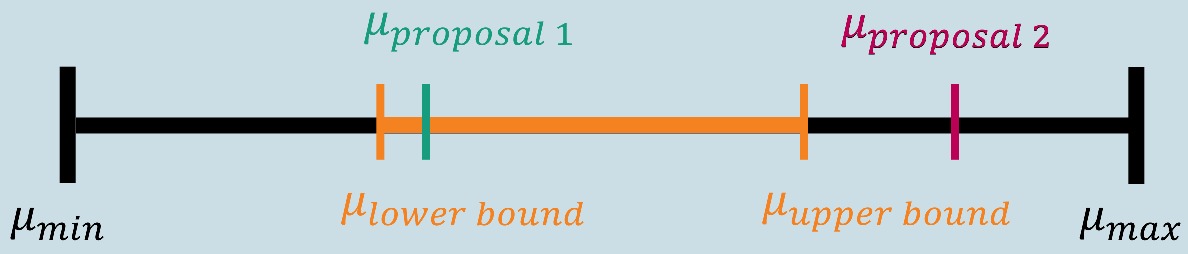

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Multicriteria Partially-Min-Regret-Robustness for a Set of Volatility Scenarios

Problem Outline

1. Several scenarios for the volatility of assets are given
2. Probability of each scenario is unknown or hard to determine
3. Weighted objective function or objective function for each scenario is not practical

How can we find an efficient portfolio that performs well in all scenarios?

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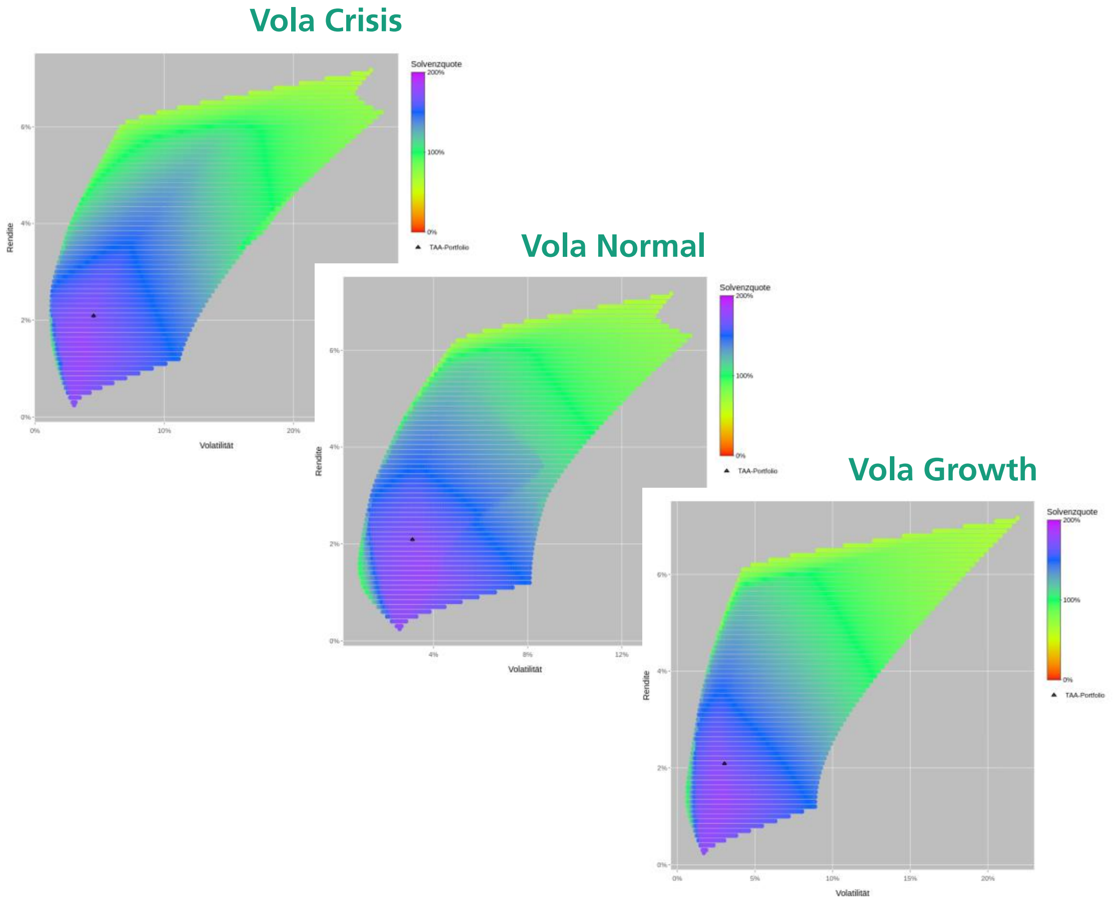

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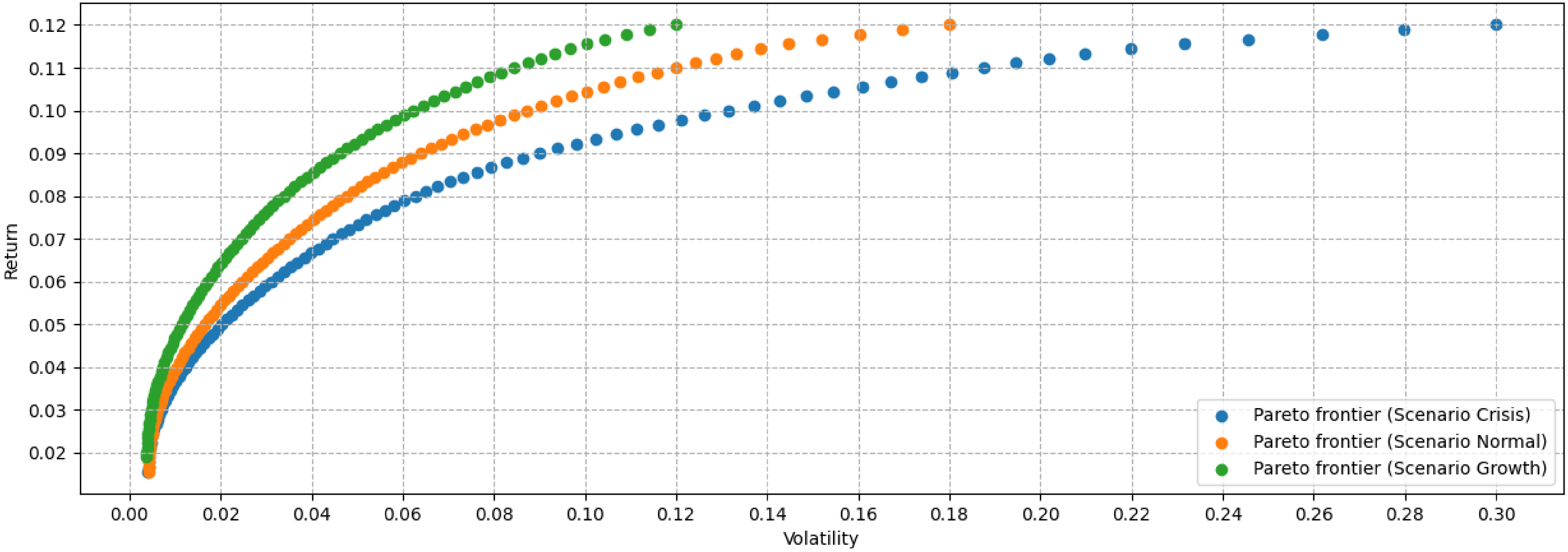

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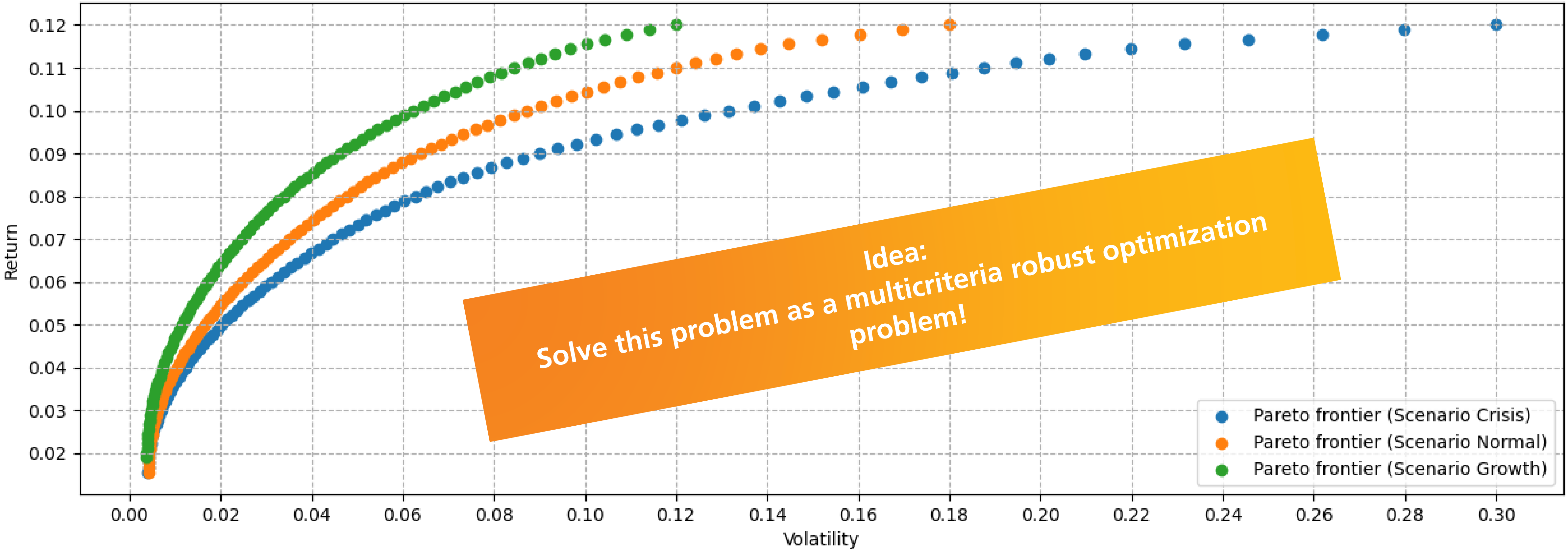

Idea:
Solve this problem as a multicriteria robust optimization problem!

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Recap: (Single-Objective) Robust Optimization

Give a set of scenarios S with scenarios ξ

Min-Max Robustness

- Minimize the worst case of the objective function under all scenarios ξ

$$\min_{x \in X} \max_{\xi \in S} f(x, \xi)$$

Min-Max Regret Robustness

- Find a solution that is closest to the best solutions for every scenario

$$\min_{x \in X} \max_{\xi \in S} \left| f(x, \xi) - \min_{\hat{x}} f(\hat{x}, \xi) \right|$$

where

- X feasible set
- f objective function
- S scenario set

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Transfer to Multicriteria Optimization ?

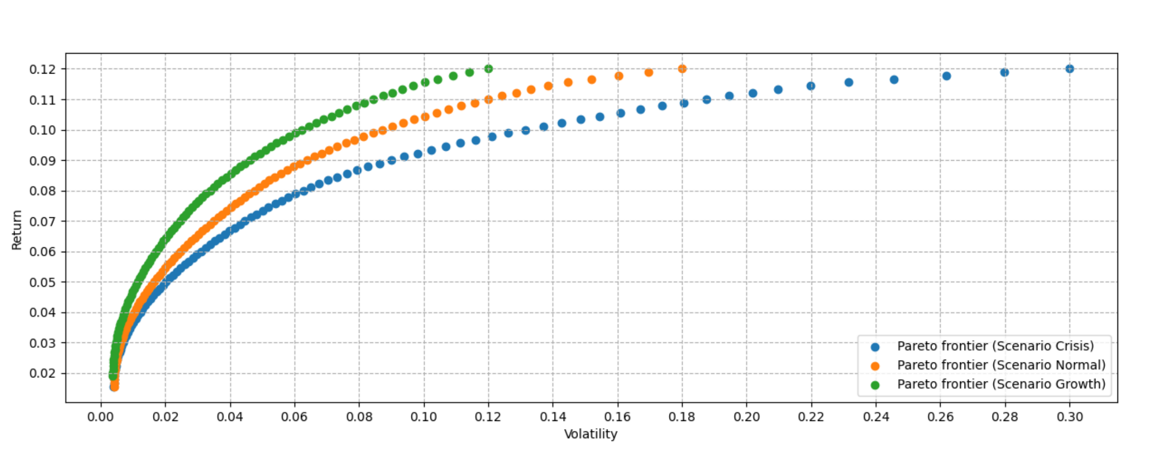
Multi-Objective Portfolio Optimization with Uncertain Volatility

Multi-Objective Min-Max Regret Robustness

- Find a solution that is objective-wise closest to the objective's best solutions for every scenario

$$\min_{x \in X} \max_{\xi \in S} \quad r(x,\xi) \qquad \text{with} \quad r_i(x,\xi) = \left| f_i(x,\xi) - \min_{\hat{x}} f_i(\hat{x},\xi) \right|$$

$$s.t. \qquad x \in [0,1]^n, \qquad \sum_{i=1}^{n} x_i = 1$$



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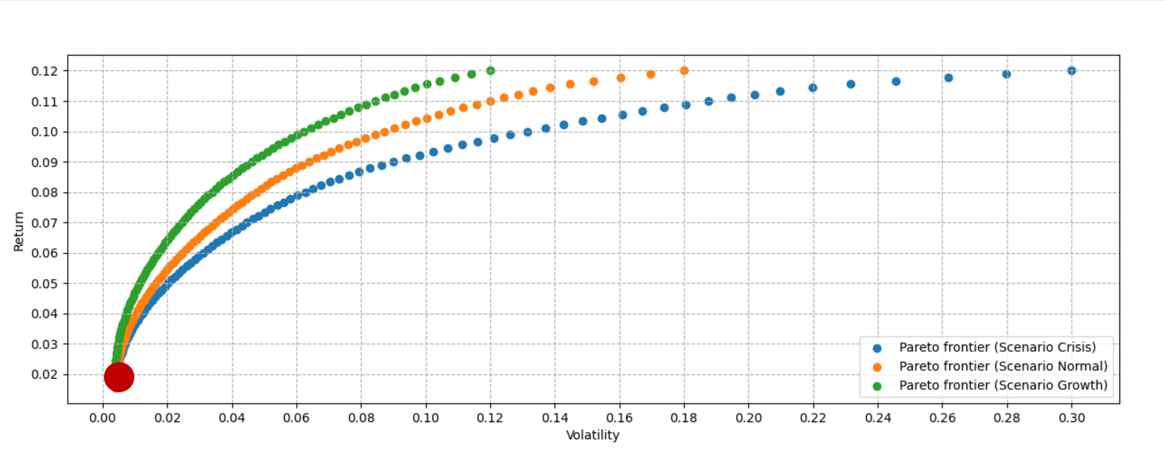

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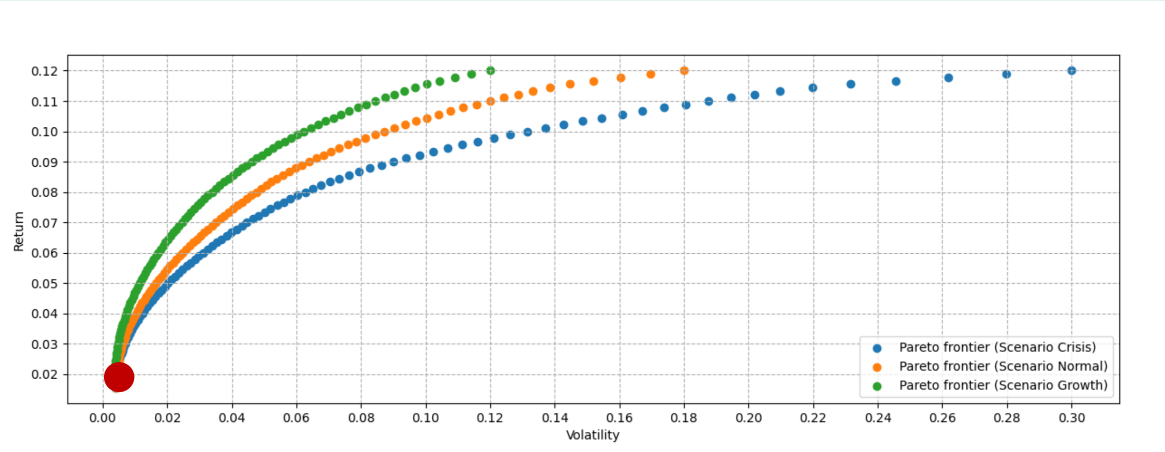

Main Limitation for Portfolio Optimization

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- Concept corresponds to Worst-Case-Robustness

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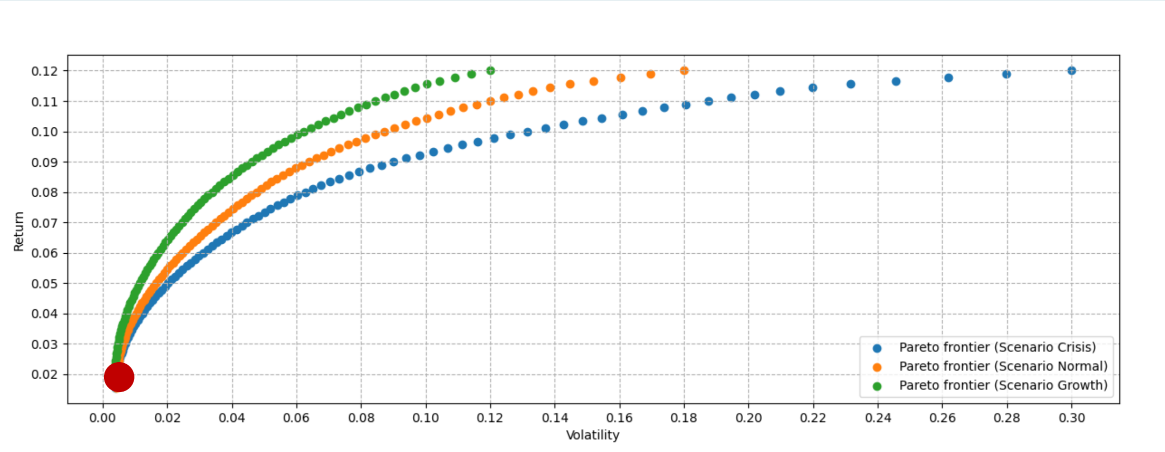

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How can we do this better?

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Multi-Objective Robust Optimization with Benchmark Solutions

Generalization of Min-Max Regret Robustness..

- Find a solution where the uncertain objective is closest to a benchmark solution for every scenario

$$\min_{x \in X} \max_{\xi \in S} \; r(x,\xi) \qquad \text{with} \qquad \mathrm{r_i}(\mathrm{x},\xi) = \begin{cases} (\| f_i(x,\xi) - \mathcal{B}(\xi) \| & , \; i = \mathscr{b} \\ f_i(x) & , \; else \end{cases}$$

where

- $\mathscr{b}$ uncertain objective
- $\mathcal{B}(\xi)$ unique benchmark solution for scenario ξ (tbd.)
- $\|.\|$ metric (tbd.)

.. in the special case of Partial Uncertainty

1. Find one benchmark solution value for each scenario with involvement of all objectives
2. Adapt the original optimization problem and only replace the uncertain objective

Practitioners can keep their framework since they just need to make a minor change

Simões, G., et al.: "Relative robust portfolio optimization with benchmark regret." Quantitative Finance, 18(12), (2018).

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How to find benchmark solutions?

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Finding Benchmark Solutions

Preliminaries

- no involvement of decision maker
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Extension of Markowitz

$$\min_{x \in X} \begin{pmatrix} -\mu^T(x) \\ \max_{\xi \in S} \left\| x^T \Sigma_\xi x - \mathcal{B}(\xi) \right\| \end{pmatrix}$$

$$s.t. \quad x \in [0,1]^n, \quad \sum_{i=1}^n x_i = 1$$

Reason why

- speed-up
- practitioners have different further objectives and can easily add them

Practitioners demand efficiency and simplicity

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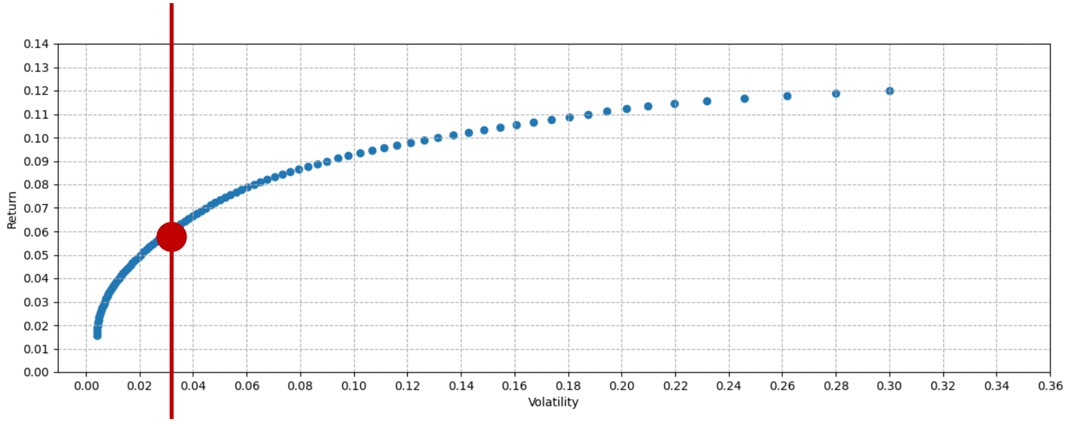

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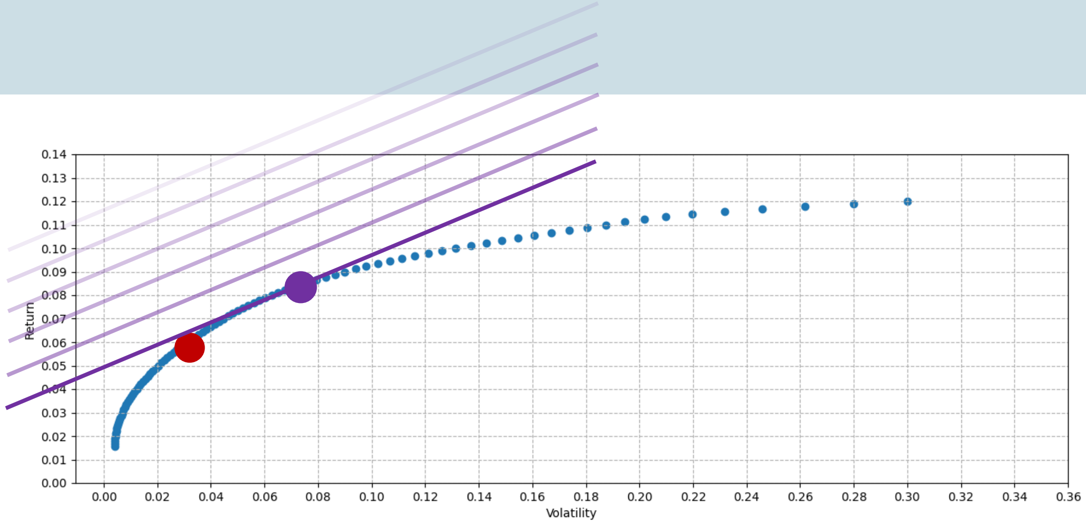

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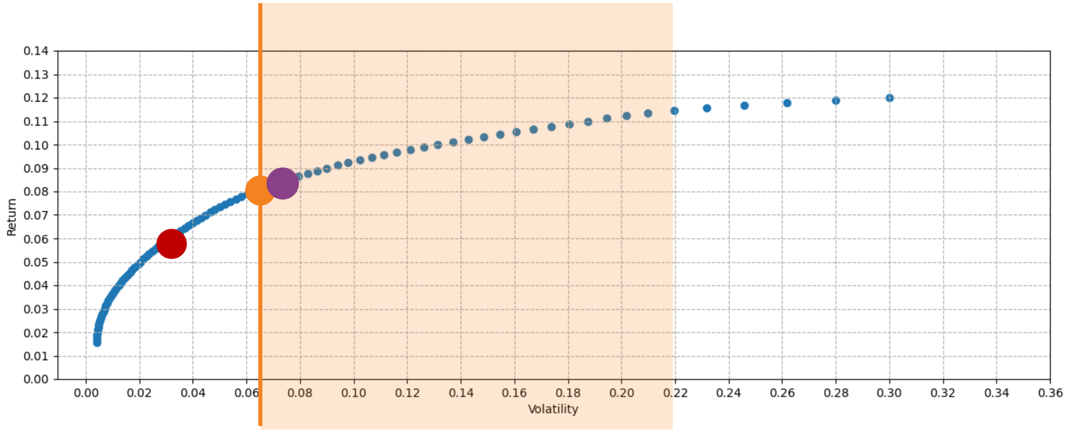
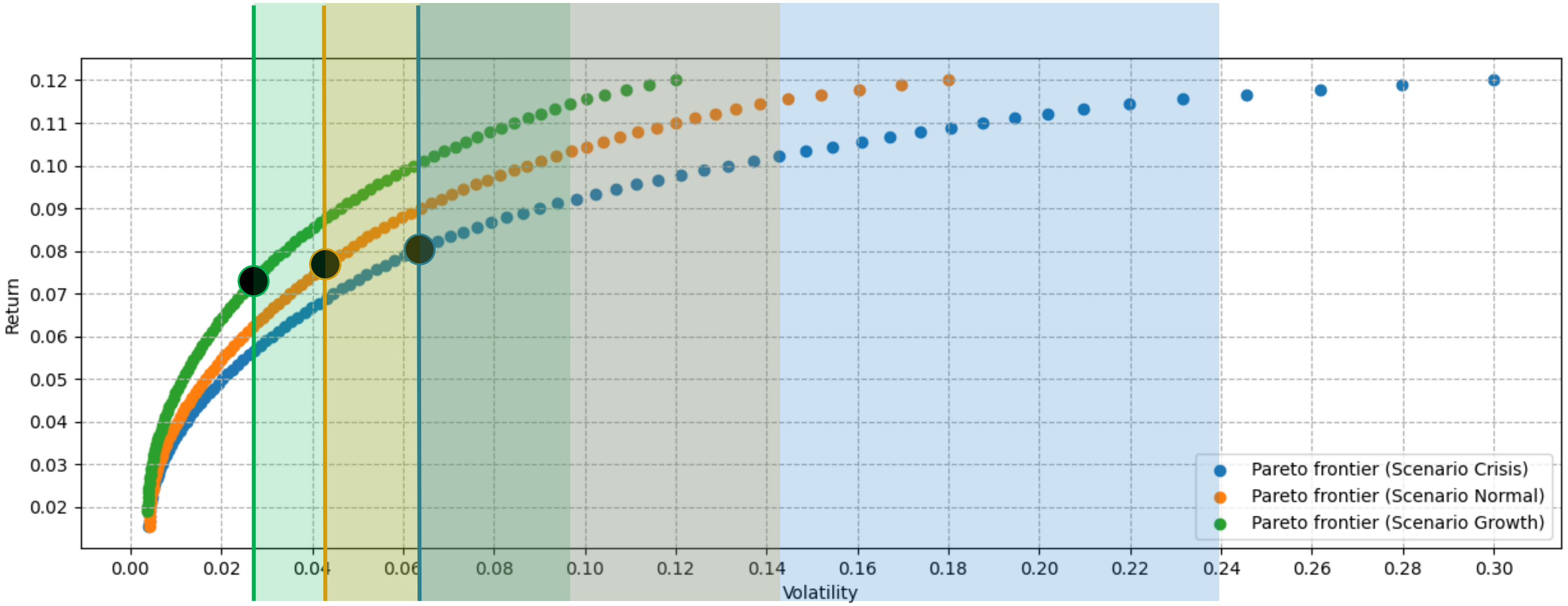
c. Lower bound of the inner 60% percentile regarding volatility of all efficient portfolios



Finding Benchmark Solutions for Each Scenario

C. Lower bound of the inner 60% percentile regarding volatility of all efficient portfolios



Growth = 2.73 %

Normal = 4.04 %

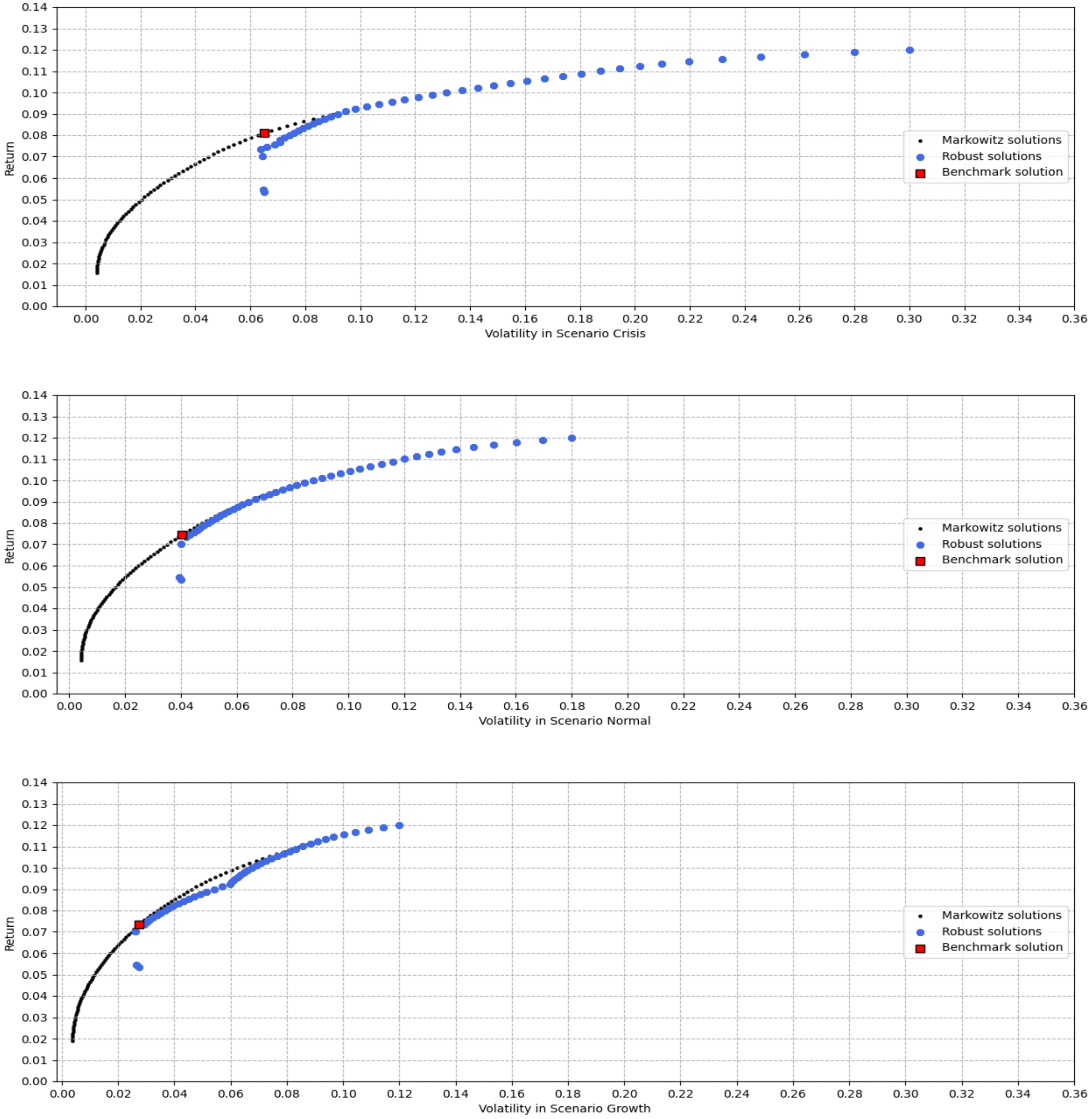
Crisis = 6.51 %

Robust Optimization Results for the Best-Performing Strategy

C. Lower bound of the inner 60% percentile regarding volatility of all efficient portfolios

Metric: 2-norm

$(f_{vola}(x,\xi) - \mathcal{B}(\xi))^2$



Markowitz solutions

Robust solutions

Benchmark solution

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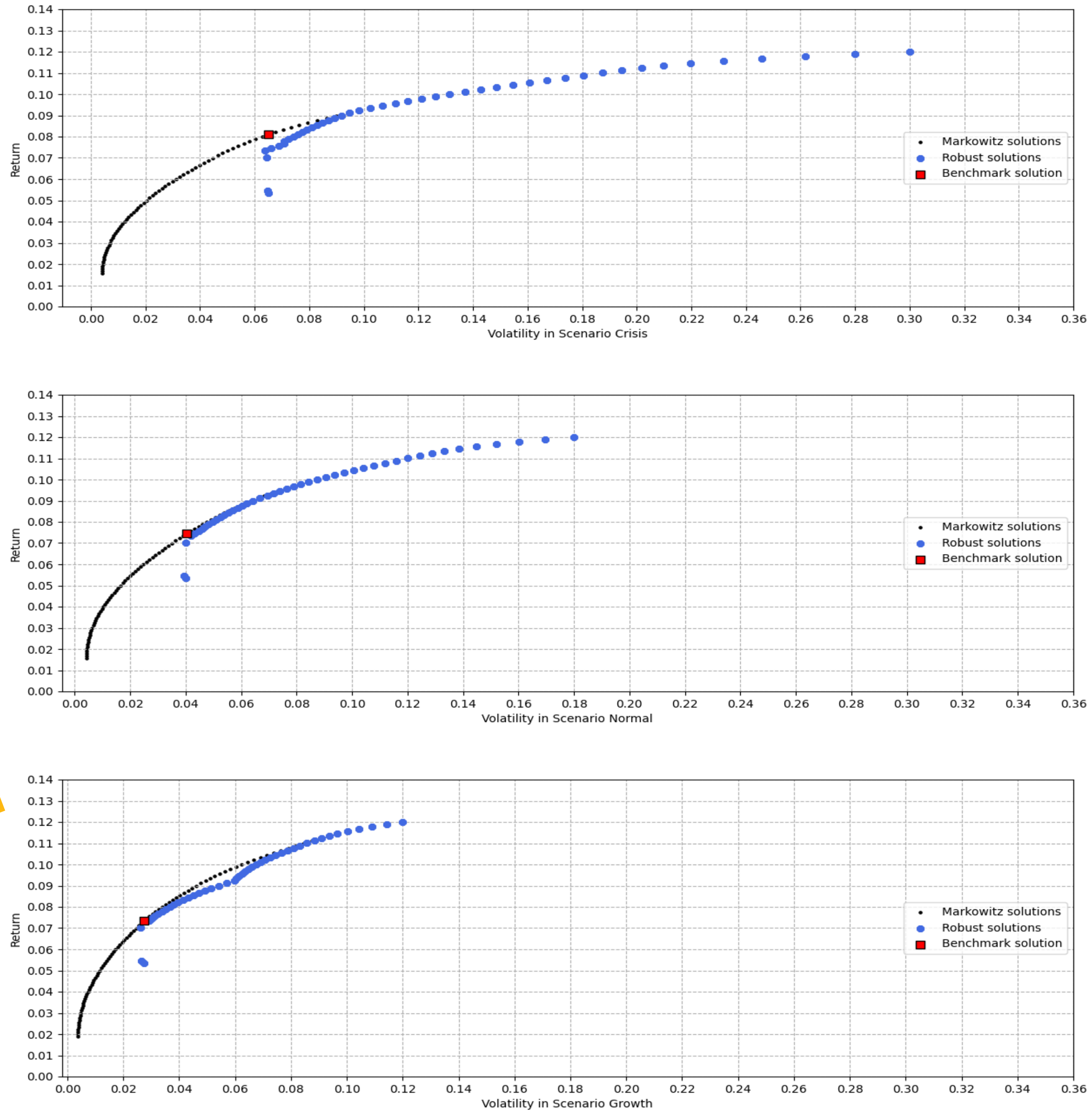

- Markowitz solutions
- Robust solutions
- Benchmark solution

Do benchmark solutions behave like bounds?

Conclusion

- Two use cases with practical relevance dealing with uncertain parameters in portfolio optimization
- Sensitivity of return parameter described by appliying multicriteria optimization algorithms
- Finding suitable solutions under varying volatility scenarios with multicriteria robust optimization
  - Introduction of a new robustness concept featuring benchmarking solutions
  - Strategy for finding user-independent benchmarking solutions
  - Evaluation of the best strategy

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Multicriteria Optimization is of high practical relevance, also in finance!