Modelling and pricing of multi-region catastrophe bonds

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ICCF24, 02.04.2024, Amsterdam

National Science Centre Poland

This work was supported by the NCN Opus 24 Grant No. 2022/47/B/HS4/0213

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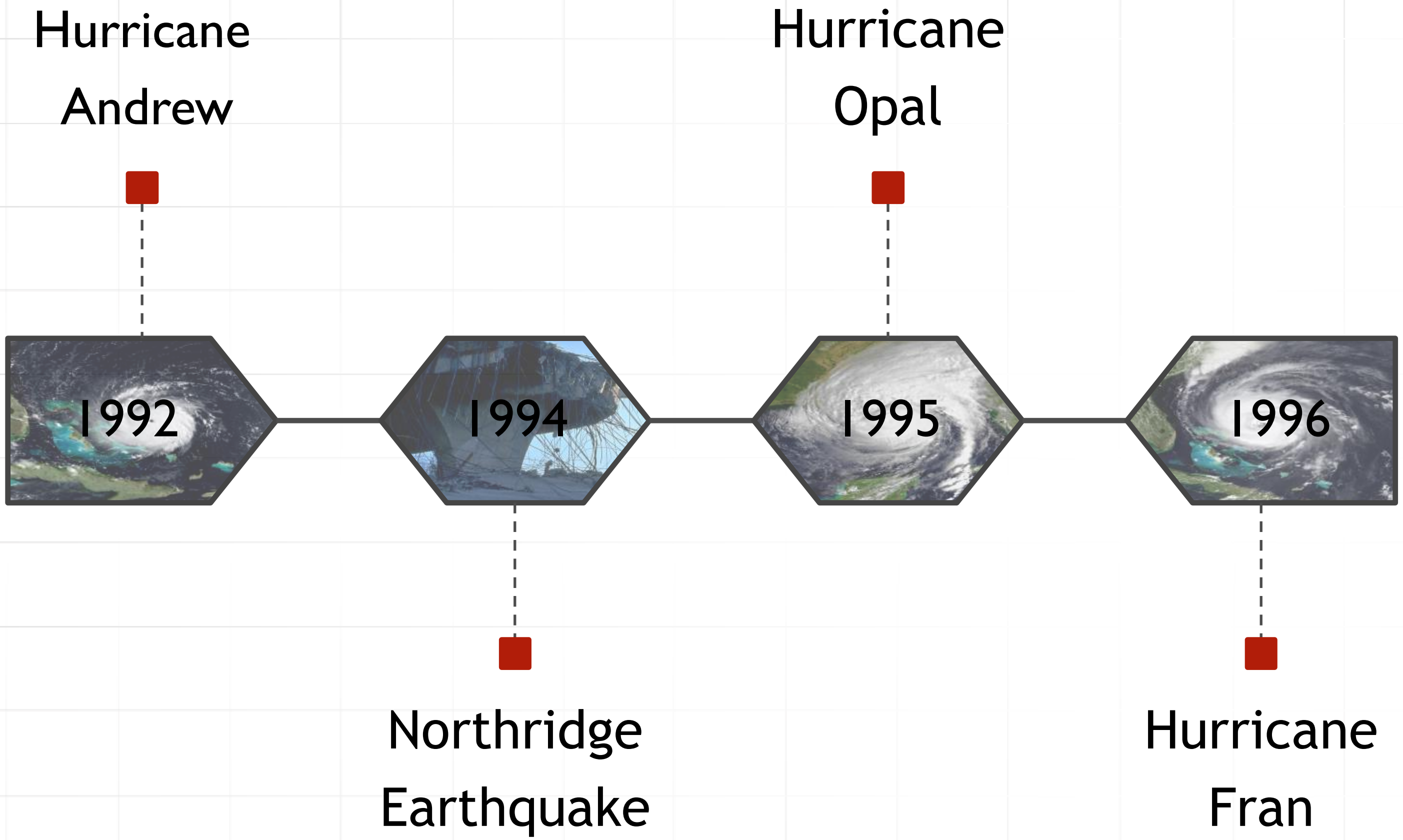
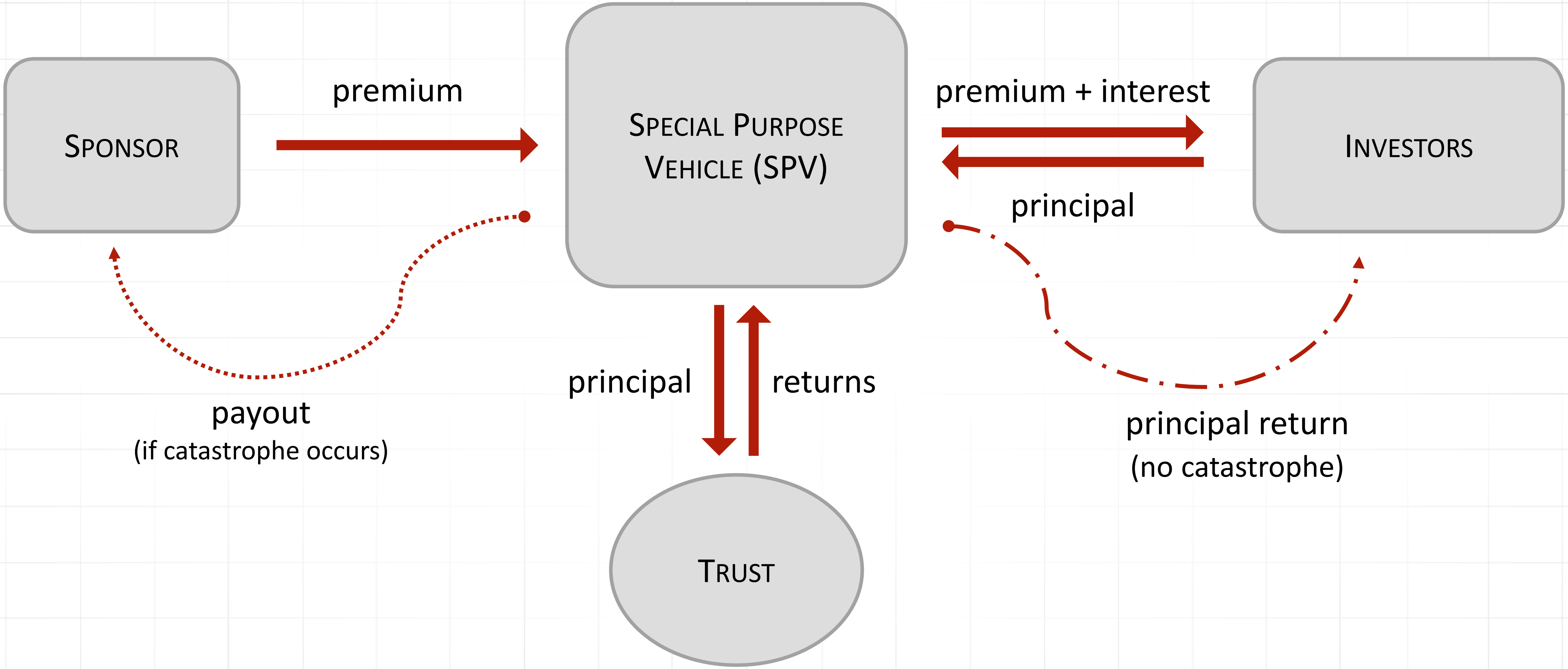
OUTLINE OF THE TALK

1. Introduction and motivation
2. Catastrophe bond pricing
3. Multi-region CAT bond
4. PCS data example



Emergence of catastrophe bonds

CAT bond construction



Multi-region aspect

Sakura Re Ltd. (Series 2022-1) – At a glance:

- **Issuer:** Sakura Re Ltd.
- **Cedent / sponsor:** Sompo International
- **Placement / structuring agent/s:** Aon Securities is sole structuring agent and bookrunner
- **Risk modelling / calculation agents etc:** AIR Worldwide
- **Risks / perils covered:** U.S. (inc. DC, Puerto Rico & Virgin Islands) & Canada named storm and earthquake
- **Size:** $150m
- **Trigger type:** Industry loss index
- **Ratings:** NR
- **Date of issue:** Dec 2022

Nature Coast Re Ltd. (Series 2023-1) – At a glance:

- **Issuer:** Nature Coast Re Ltd.
- **Cedent / sponsor:** Safepoint Insurance Company
- **Placement / structuring agent/s:** Aon Securities is sole structuring agent and bookrunner
- **Risk modelling / calculation agents etc:** AIR Worldwide
- **Risks / perils covered:** U.S. named storm (Florida, Louisiana)
- **Size:** $195m
- **Trigger type:** Indemnity
- **Ratings:** NR
- **Date of issue:** Nov 2023

Source: https://www.artemis.bm/deal-directory

Pricing methods

in practice – done by specialized companies, expected loss (EL) and probability of first loss (PFL) are provided for the investors

↓

first no-arbitrage pricing of CAT bonds, Baryshnikov, Mayo and Taylor (1998)

↓

further developments

Lee and Yu (2002), Vaugirard (2003), Burnecki and Kukla (2003), Ma and Ma (2013), Nowak and Romaniuk (2013)

↓

problem of multi-peril and multi-region catastrophes

Reshetar (2008), Hofer et al. (2020)

General pricing formula

The general pricing formula at time t for CAT bonds can be written as:

$$V_t = e^{-r(T-t)}\mathbb{E}_{\mathbb{P}}[P(T)|\mathcal{F}_t],$$

where $P(T)$ is the payoff at maturity T of the bond, $\mathbb{E}_{\mathbb{P}}$ denotes the expectation under the real-world measure, r is a constant interest rate over $[0, T]$ and $\mathcal{F}_t$ is the filtration up until time t.

Aggregate loss process

Aggregate loss process (ALP) is a stochastic process $\{L(t), t > 0\}$ that describes the total amount of losses in time. It is defined as:

$$L(t) = \sum_{k=1}^{N(t)} X_k \,,$$

where $\{N(t), t > 0\}$ is a loss counting process and the loss amounts are i.i.d. positive random variables $\{X_k, k \in \mathbb{N}\}$ with $\mathbb{E}[X_k] < \infty$.

We also assume that loss counting process and loss amounts are independent.

Zero-coupon (ZC) CAT bond

The payoff of a zero-coupon CAT bond per unit nominal is given by:

$$P_{ZC}(T) = \begin{cases} 1 \text{ if } L(T) < D, \\ \text{c if } L(T) \geq D, \end{cases}$$

where T is the term of the bond, D is a specified threshold level triggering the bond and $0 \leq \mathrm{c} \leq 1$ is a constant recovery rate.

The arbitrage-free price of a zero-coupon CAT bond is given by:

$$V_0 = e^{-rT}\mathbb{E}_{\mathbb{P}}\left[\mathbb{I}_{L(T)<D} + \mathrm{c}\mathbb{I}_{L(T)\geq D}\right] = e^{-rT}[\mathrm{c} + (1-\mathrm{c})\mathbb{P}(L(T) < D)].$$

Multi-Region ZC CAT BOND

Let $\mathbf{L} = (L_1, L_2, \dots, L_n)$ be a multi-dimensional ALP process, where $\{L_i(t), t > 0\}$ denotes ALP resulting from i-th region, for $i = 1, \dots, n$.

The payoff of a multi-region zero-coupon CAT bond per unit nominal is given by:

$$P_{ZC}^{MR}(T) = \begin{cases} 1 \text{ if } \cap_{i=1}^{n} \{L_i(T) < D_i\}, \\ c \text{ if } \cup_{i=1}^{n} \{L_i(T) \geq D_i\}, \end{cases}$$

where T is the term of the bond, $D_1, D_2, \dots, D_n$ are specified threshold levels for the corresponding ALPs $L_1, L_2, \dots, L_n$ and $0 \leq \mathrm{c} \leq 1$ is a constant recovery rate.

Two-region ZC CAT BOND

In case of a two-region bond, we can define the payoff as:

$$P(T) = \begin{cases} 1 \text{ if } (L_1(T) < D_1 \ \wedge L_2(T) < D_2), \\ c \text{ if } (L_1(T) \geq D_1 \ \vee L_2(T) \geq D_2). \end{cases}$$

The price of a two-region zero-coupon CAT bond is given by:

$$V_0 = e^{-rT}[c + (1-c)\mathbb{P}(L_1(T) < D_1 \wedge L_2(T) < D_2)].$$

Proposed models

Independent losses	Proportionally split common losses	Dependent common losses
We assume all losses are mutually independent.	Losses that are common for both regions are shared with a given proportion $p, 0 \leq p \leq 1$.	Losses that are common for both regions are correlated with given correlation coefficient.
$\begin{cases} S_1(t) = \sum_{i=1}^{N^{(1)}(t)} X_i \\ S_2(t) = \sum_{i=1}^{N^{(2)}(t)} Y_i \end{cases}$	$\begin{cases} S_1(t) = \sum_{i=1}^{N^{(1)}(t)} X_i + p \sum_{i=1}^{N^{(3)}(t)} Z_i \\ S_2(t) = \sum_{i=1}^{N^{(2)}(t)} Y_i + (1-p) \sum_{i=1}^{N^{(3)}(t)} Z_i \end{cases}$	$\begin{cases} S_1(t) = \sum_{i=1}^{N^{(1)}(t)} X_i^{(1)} + \sum_{i=1}^{N^{(3)}(t)} X_i^{(2)} \\ S_2(t) = \sum_{i=1}^{N^{(2)}(t)} Y_i^{(1)} + \sum_{i=1}^{N^{(3)}(t)} Y_i^{(2)} \end{cases}$

Normal approximation

The price of a two-region zero-coupon CAT bond can be approximated as:

$$V_0 \approx V_0^{approx} = e^{-rT}[c + (1-c)\mathbb{P}(N_1 < D_1 \wedge N_2 < D_2)],$$

where $\boldsymbol{N} = (N_1, N_2)$ is a random vector with bivariate normal distribution with mean and covariance matrix:

$$\mu = \begin{pmatrix} ES_1(T) \\ ES_2(T) \end{pmatrix},$$

$$\Sigma = \begin{pmatrix} Var\, S_1(T) & Cov(S_1(T), S_2(T)) \\ Cov(S_1(T), S_2(T)) & Var\, S_2(T) \end{pmatrix}.$$

Figure 1 Location of analysed pairs of states in the USA - Oklahoma and Texas, Illinois and Kentucky.

Oklahoma & Texas

- There were 85 catastrophes in Oklahoma and 163 in Texas, 44 of them occurred in both states.
- For model with proportionally split common losses, we set the proportion $p = 0.35$.
- The Spearman correlation coefficient was $\rho = 0.3116$, for model with correlated common losses.

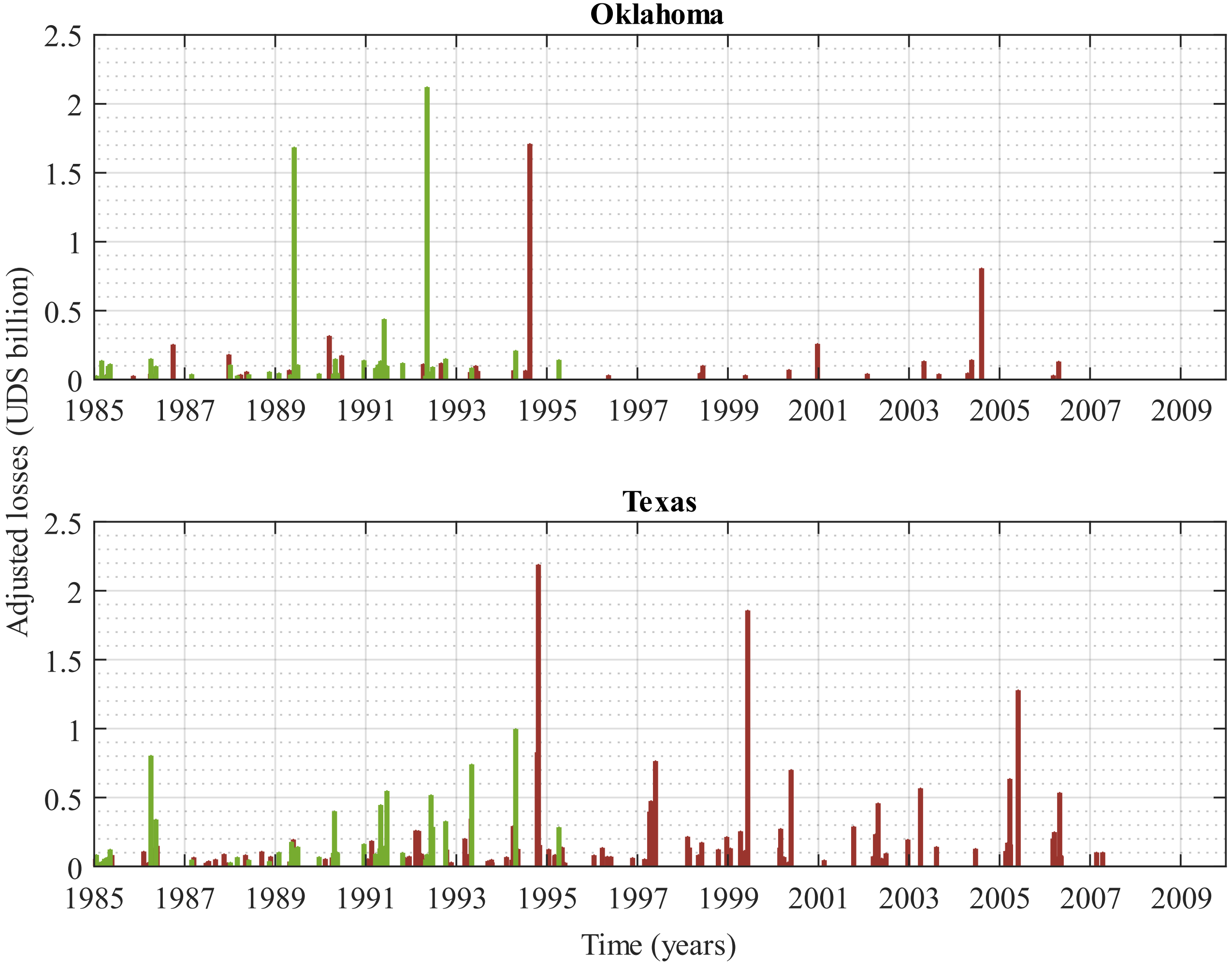


Figure 2 Adjusted losses in Oklahoma and Texas. Losses from common catastrophes are presented in green, from unrelated ones are in red.

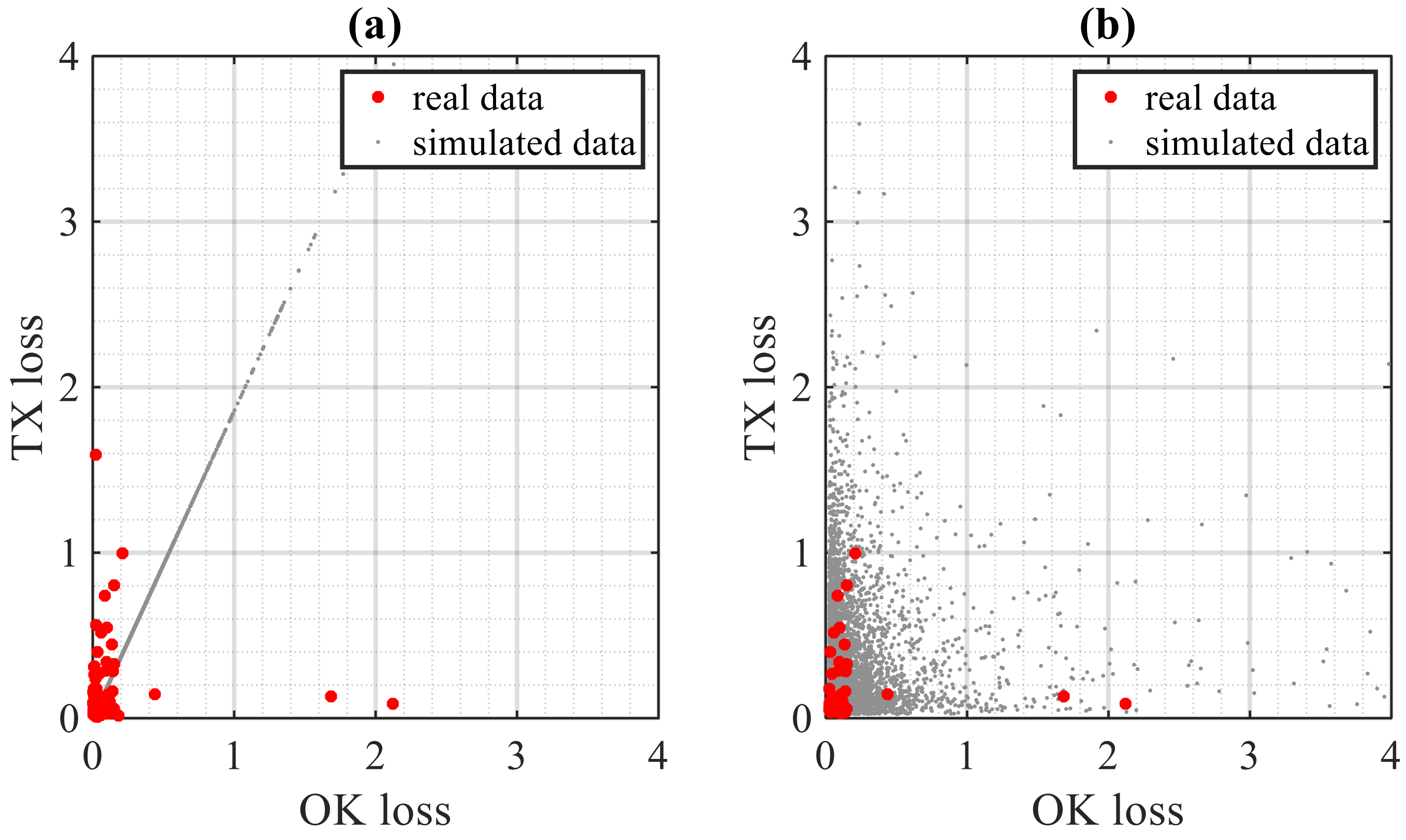


Figure 3 Comparison of real common losses in Oklahoma and Texas and common losses simulated in (a) proportionally split model and (b) dependent model.

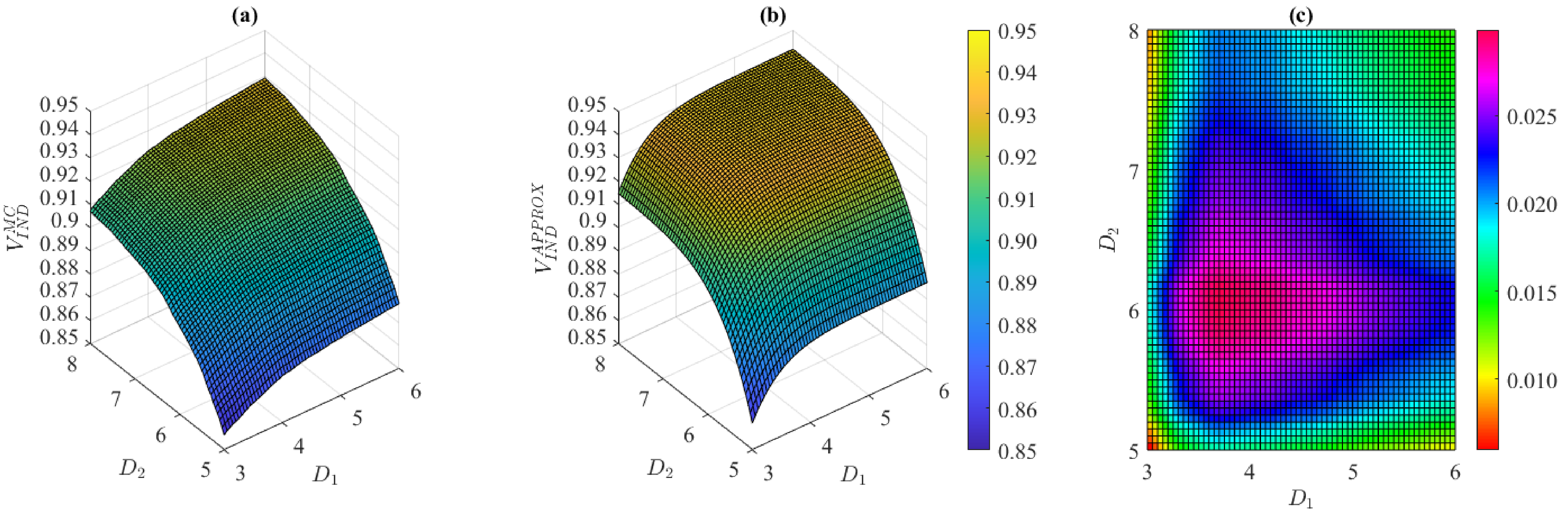


Figure 4 Model with independent losses. Zero-coupon CAT bond for Oklahoma and Texas:
(a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

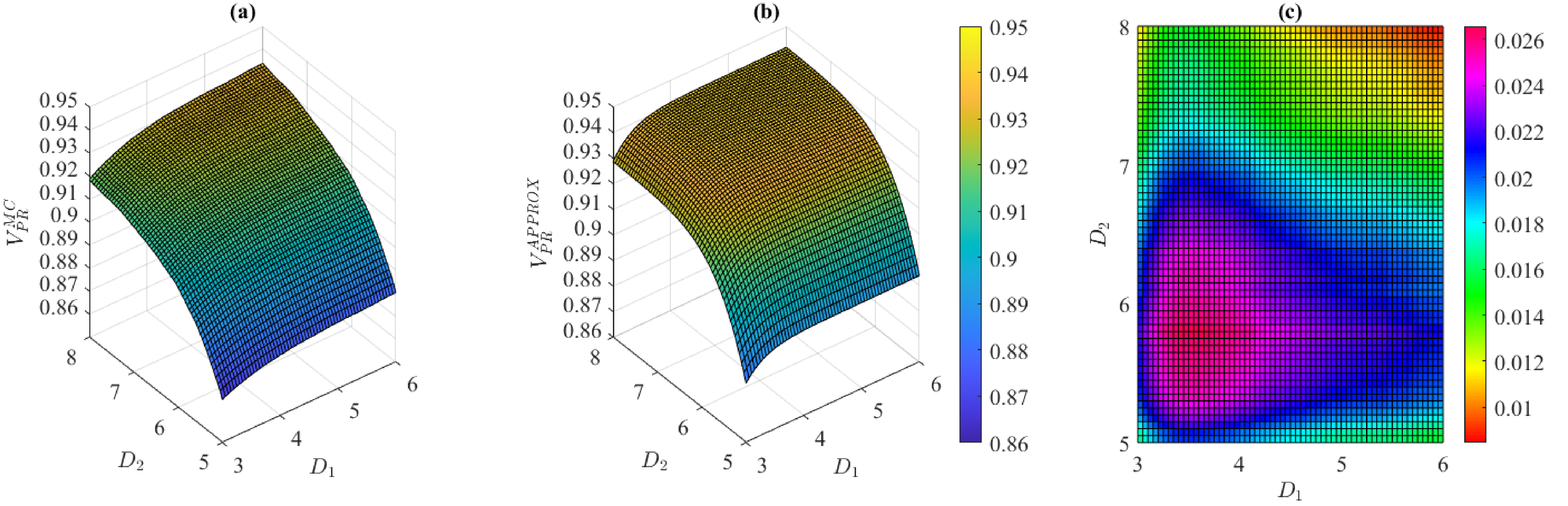


Figure 5 Model with proportionally split common losses. Zero-coupon CAT bond for Oklahoma and Texas: (a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

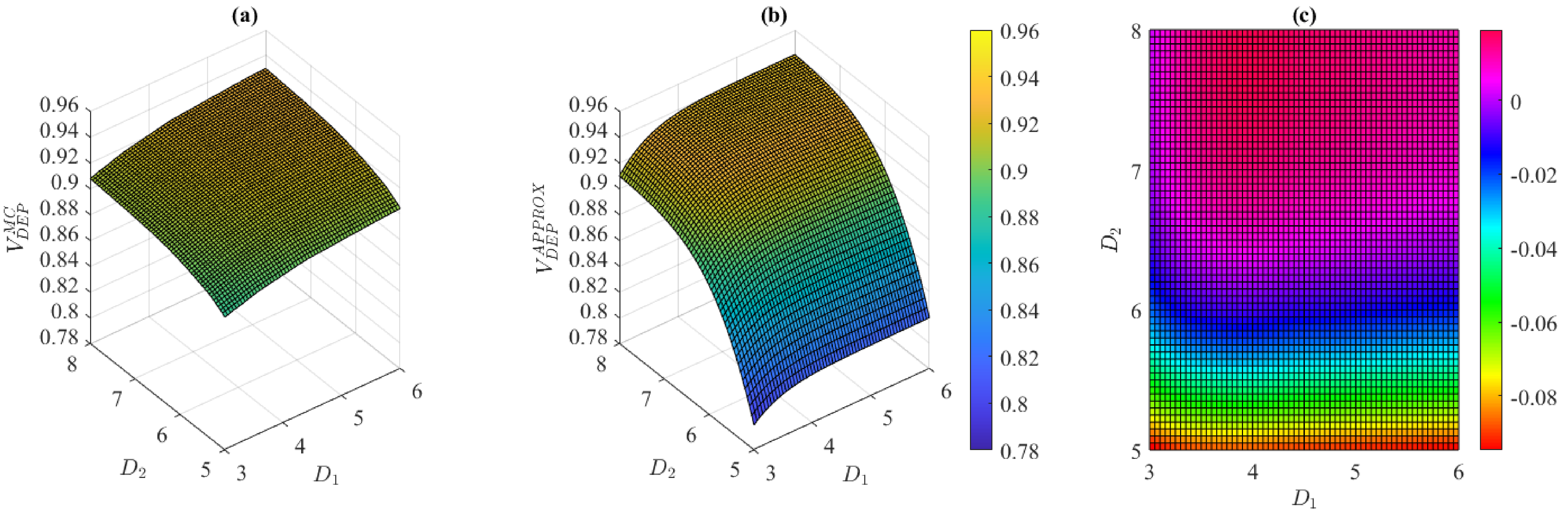


Figure 6 Model with correlated common losses. Zero-coupon CAT bond for Oklahoma and Texas: (a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

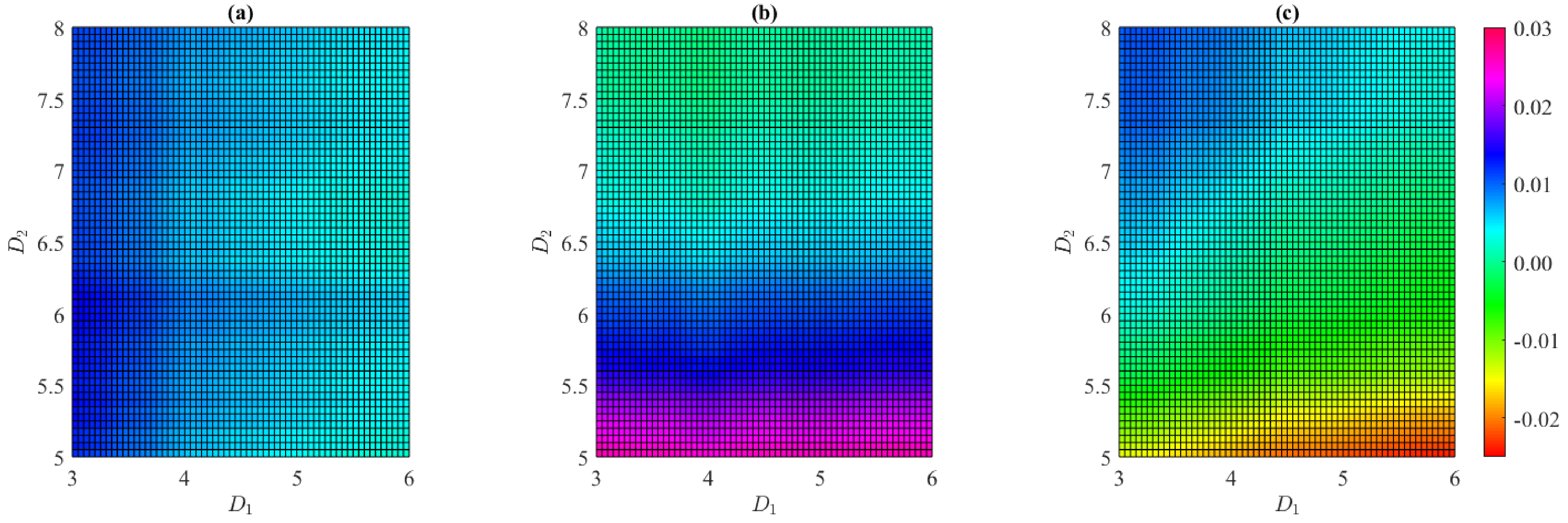


Figure 7 Differences between prices of zero-coupon CAT bond for Oklahoma and Texas obtained from models with: (a) proportionally split common losses and independent losses, (b) correlated common losses and independent losses, (c) proportionally split and correlated common losses.

Illinois & Kentucky

- There were 111 catastrophes in Illinois and 45 in Kentucky, 27 of them occurred in both states.
- For model with proportionally split common losses, we set the proportion $p = 0.56$.
- The Spearman correlation coefficient was $\rho = 0.2244$, for model with correlated common losses.

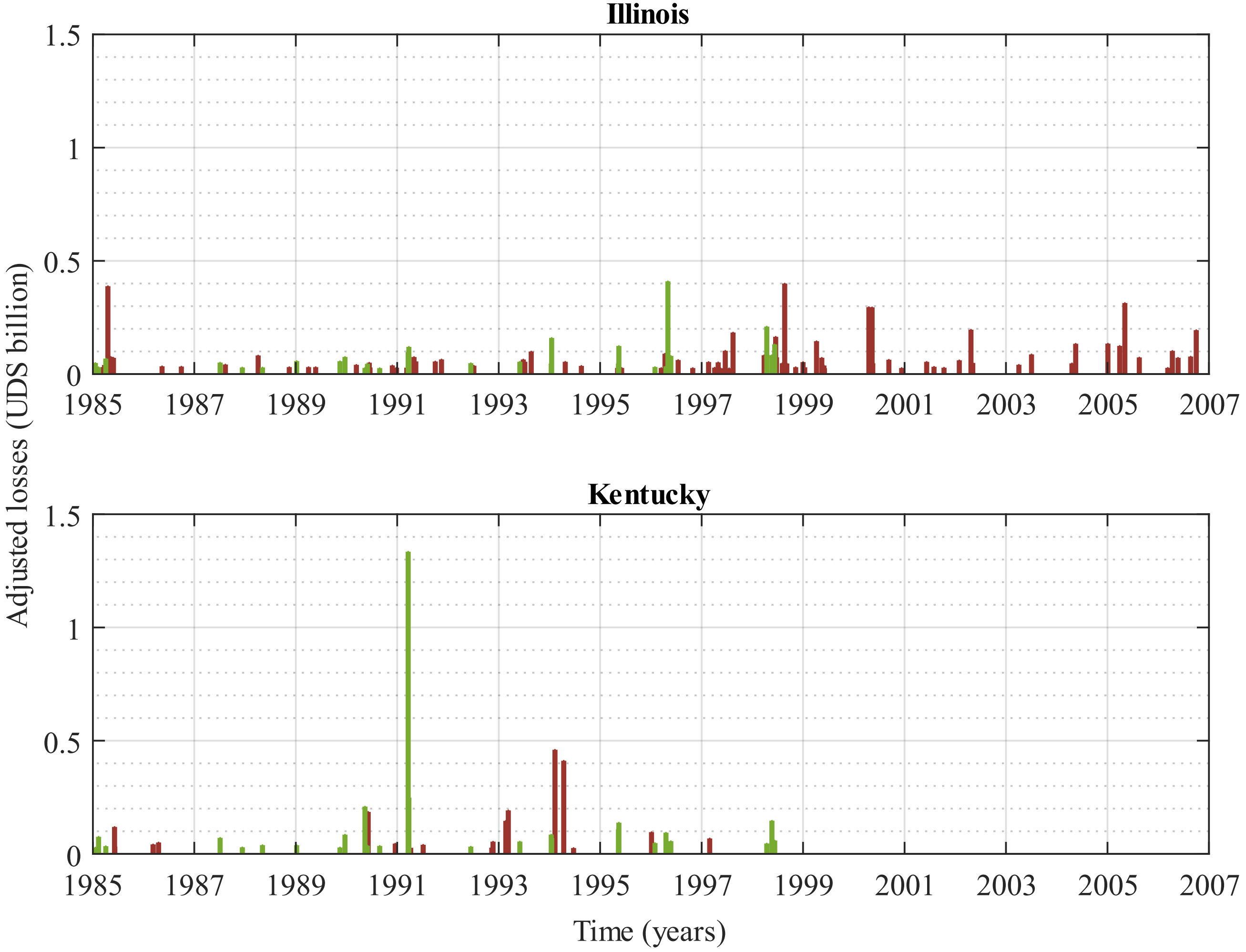


Figure 8 Adjusted losses in Illinois and Kentucky. Losses from common catastrophes are presented in green, from unrelated ones are in red.

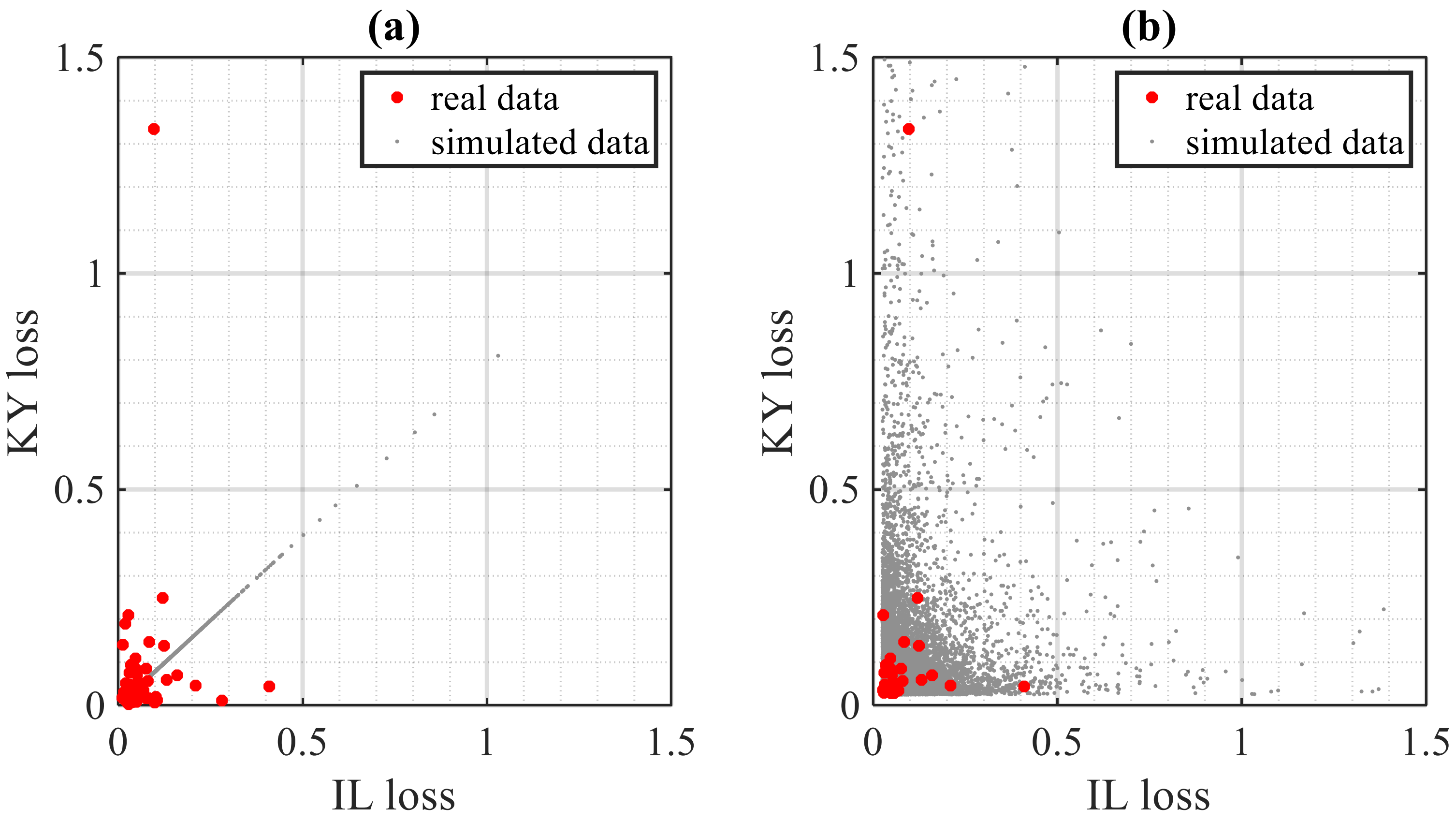


Figure 9 Comparison of real common losses in Illinois and Kentucky and common losses simulated in (a) proportionally split model and (b) dependent model.

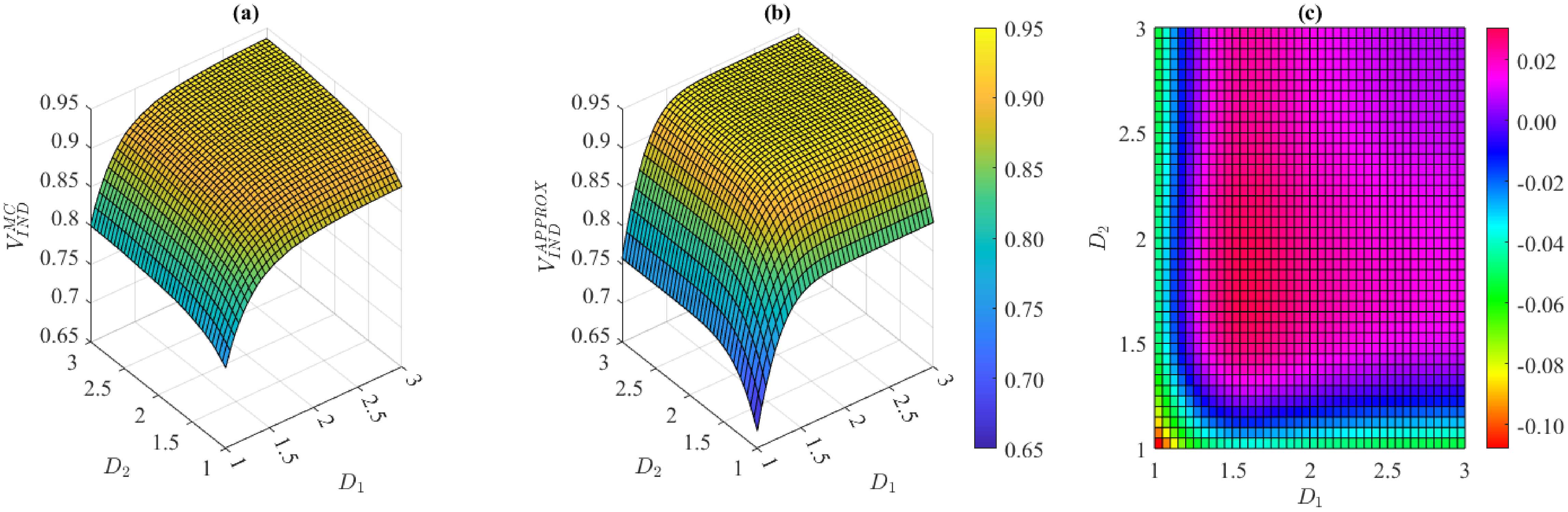


Figure 10 Model with independent losses. Zero-coupon CAT bond for Illinois and Kentucky: (a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

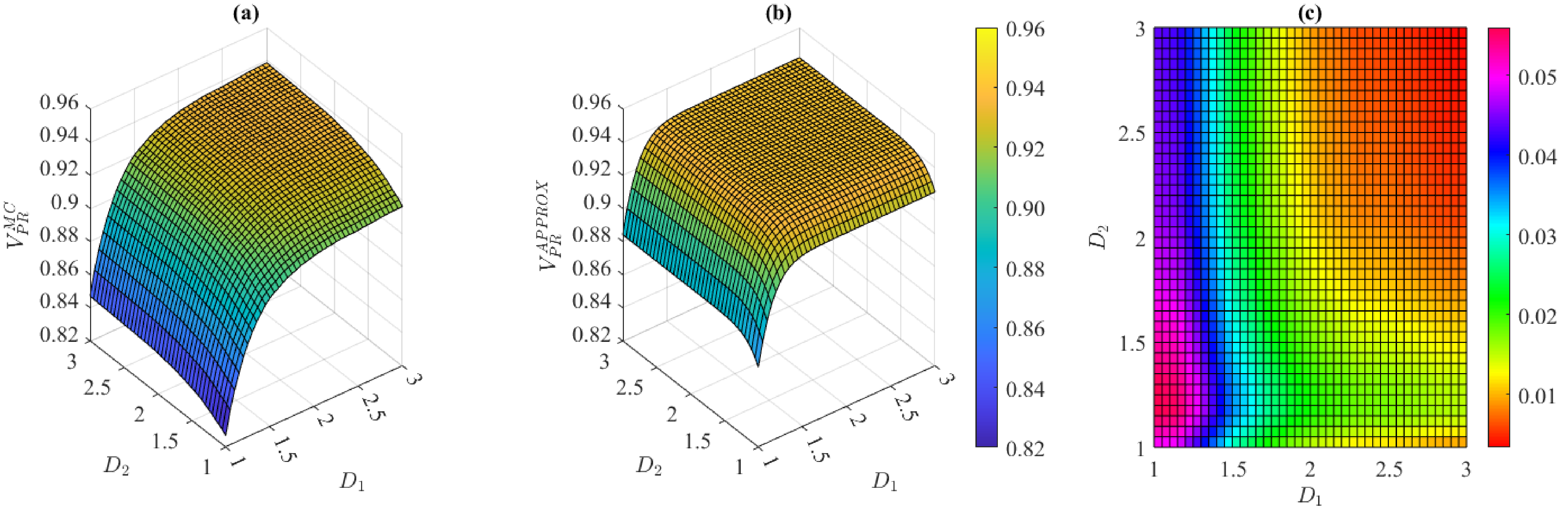


Figure 11 Model with proportionally split common losses. Zero-coupon CAT bond for Illinois and Kentucky: (a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

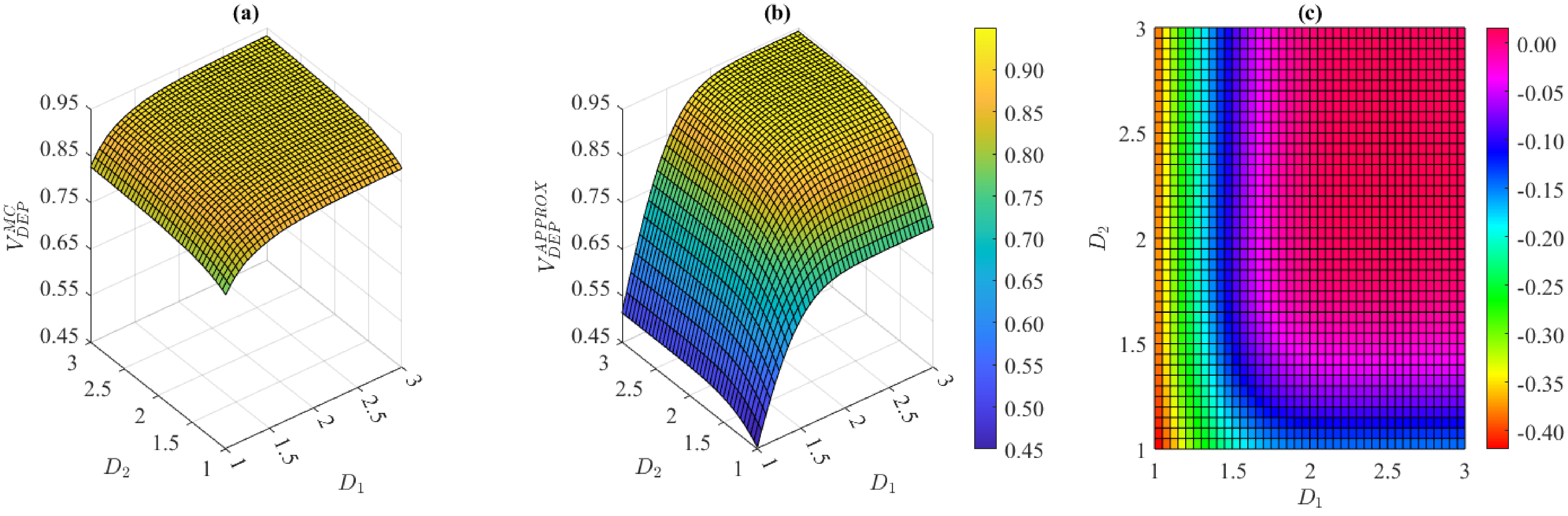


Figure 12 Model with correlated common losses. Zero-coupon CAT bond for Illinois and Kentucky: (a) price from Monte Carlo simulations, (b) normal approximation and (c) relative difference.

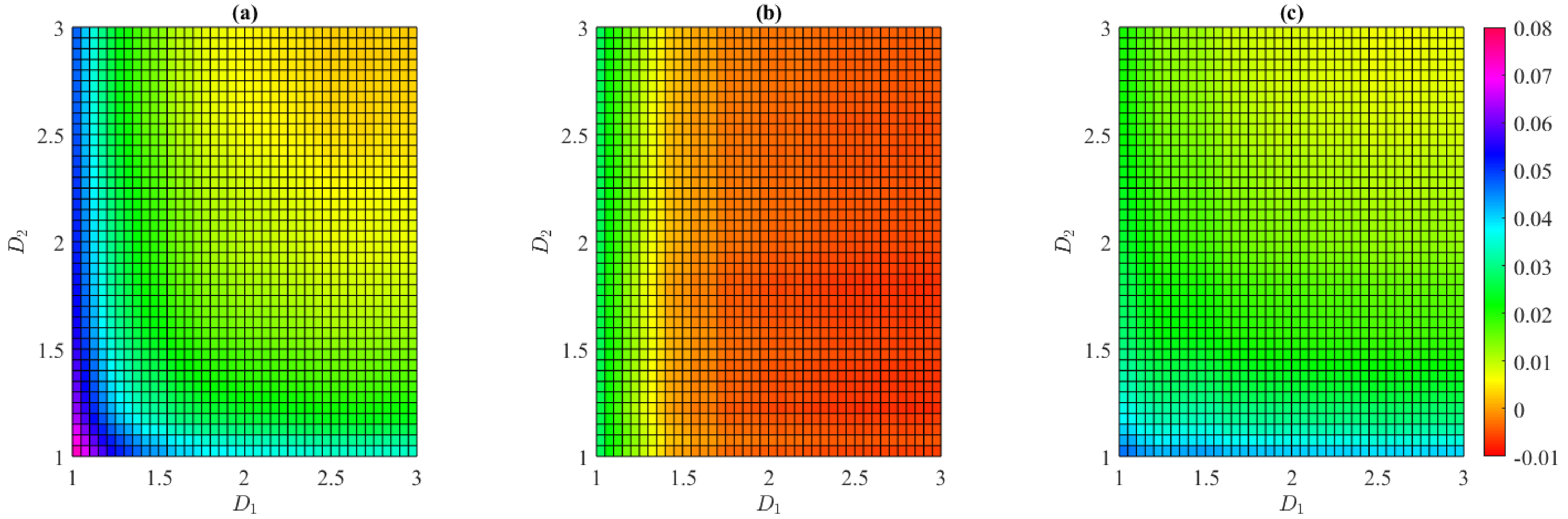


Figure 13 Differences between prices of zero-coupon CAT bond for Illinois and Kentucky obtained from models with: (a) proportionally split common losses and independent losses, (b) correlated common losses and independent losses, (c) proportionally split and correlated common losses.

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Thank you
for your attention!