Mini-symposium on Climate risk and financial risk impact

- Aurélien Alfonsi (Ecole des Ponts, France): "Risk valuation of quanto derivatives on temperature and electricity."
- Florian Bourgey (Bloomberg, USA): "Climate risk assessment of a large-sized credit portfolio"
- Elisa Ndiaye (Ecole Polytechnique and BNP Paribas, France): "Optimal business model adaptation plan for a company under a transition scenario"
- Jörg Müller (Chemnitz, Germany): "Credit value-at-risk in the context of ESG"

Special thanks to Ying Jiao (ISFA, Lyon) for having organized this session.

Risk valuation of quanto derivatives on temperature and electricity

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ICCF 2024
joint work with Nerea Vadillo Fernandez (AXA Climate)

Ecole des Ponts

4th April, 2024

Introduction

- Climate change leads to a growing demand for risk transfer instruments, in order to hedge against its consequences.
- Weather derivatives emerged in the 1990s to deal with the risk on temperature, drought, etc.
- Slowdown of the weather derivatives market after the subprime crisis, that is also due to the birth on new hybrid derivatives combining weather and energy: the quantos.

Introduction

- Climate change leads to a growing demand for risk transfer instruments, in order to hedge against its consequences.
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Need for modelling dependence :
in this talk, for electricity spot price $(e^{X_t})_{t\geq 0}$ and temperature $(T_t)_{t\geq 0}$

What are quanto derivatives?

Contract structure:

- A weather index : we consider here the daily temperature T_t,
- An energy price index : we consider here the daily average spot price $S_t = e^{X_t}$,
- A payoff depending on the product of two payoff functions f_S and f_T

$$Payoff := \sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t).$$

Main interest: hedge against both

- volumetric risk (e.g. higher electricity demand due to heating/cooling),
- price risk.

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OTC Market, no liquid assets on temperature derivatives:

- We propose a real world model.
- Risk valuation under historical probability.

Outline of The Talk

Data for our study

From 5th January 2015 to 31st December 2018

Markets	Energy data	Weather data
France	Day ahead prices from ENTSO-E Transparency Platform	Paris Charles de Gaulle
North Italy	PUN from Gestore Mercati Energetici (GME)	Milano Linate

Outline

The joint model for $(X_t, T_t)_{t\geq 0}$

A coupled model for electricity spot (log)price $(X_t)_{t\geq 0}$ and average temperature $(T_t)_{t\geq 0}$

$$\begin{cases} d(X_t - \mu_X(t)) & = -\kappa_X(X_t - \mu_X(t)) + \lambda\sigma_T dW_t^T + dL_t^X \\ d(T_t - \mu_T(t)) & = -\kappa_T(T_t - \mu_T(t)) + \sigma_T dW_t^T \end{cases} \qquad \text{(ETM)}$$

where

- $\mu_X, \mu_T : \mathbb{R}_+ \to \mathbb{R}$ represent the trend and seasonality component,
- $\kappa_X, \kappa_T > 0$ correspond to the mean-reverting (or autoregressive) behaviour,
- W^T Brownian notion, L^X NIG Lévy noise, independent,
- $\lambda \in \mathbb{R}$ dependence parameter.

The joint model for $(X_t, T_t)_{t\geq 0}$

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Advantages

- Convincing marginals on both underlyings.
- Integrates dependence structure.
- Maintains autoregressive behaviour.
- Tractable model:
  - Ease to estimate,
  - Semi explicit formulas for useful expectations.

Marginal model for log spot electricity price $(X_t)_{t\geq 0}$

A model inspired by Benth and Benth (2004)

$$d(X_t - \mu_X(t)) = -\kappa_X(X_t - \mu_X(t)) + \lambda\sigma_T dW_t^T + dL_t^X$$

where

- The deterministic seasonality function μ_X

$$\mu_X(t) = \beta_0^X t + \alpha_1^X \sin(\xi t) + \beta_1^X \cos(\xi t) + \alpha_{DoW(t)}^{X,DoW}$$

  where $\xi = \frac{2\pi}{365}$ and $DoW(t) = \lfloor \frac{t}{\Delta} \rfloor$ mod $p = 7$,
- $\kappa_X > 0$ corresponds to the mean-reverting parameter,
- L^X is a Normal Inverse Gaussian Lévy process of parameters $(\alpha^X, \beta^X, \delta^X, m^X)$, centered $(m^X + \delta^X \frac{\beta^X}{\gamma^X} = 0.)$.

Marginal model for log spot electricity price $(X_t)_{t\geq 0}$

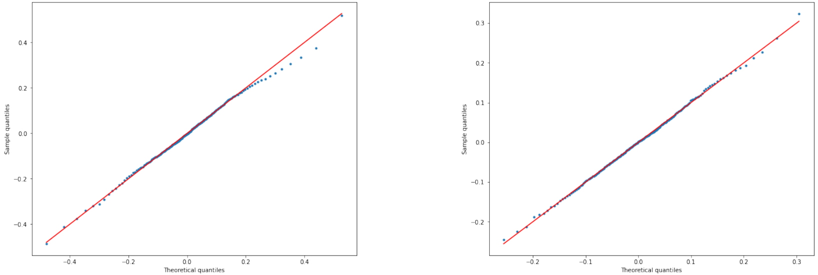


Figure: Quantile quantile plots for residuals compared with a theoretical quantiles of a normal inverse gaussian distribution for French energy (left) and North Italian Energy (right).

Marginal model for average temperature $(T_t)_{t\geq 0}$

A well established model developed by Benth et al. (2007)

$$d(T_t - \mu_T(t)) = -\kappa_T(T_t - \mu_T(t)) + \sigma_T dW_t^T$$

where

- The deterministic trend and seasonality function μ_T:

$$\mu_T(t) = \alpha_0^T + \beta_0^T t + \alpha_1^T \sin(\xi t) + \beta_1^T \cos(\xi t), \text{ where } \xi = \frac{2\pi}{365}.$$

- κ_T corresponds to the mean-reverting parameter,
- W^T is a Brownian motion and $\sigma_T > 0$ to the standard deviation of the noise.

Marginal model for average temperature $(T_t)_{t\geq 0}$

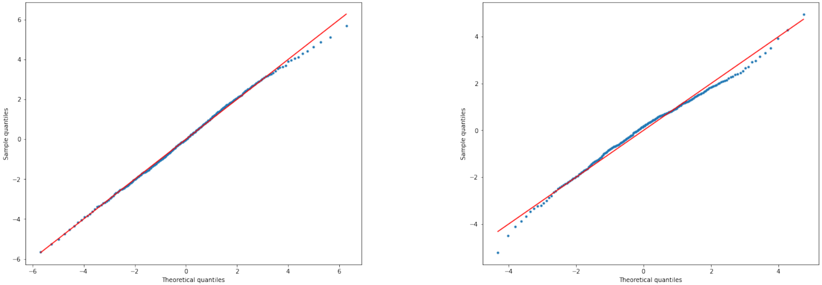


Figure: Quantile quantile plots for residuals compared with a theoretical quantiles of a normal distribution for Paris temperatures (left) and Milan temperatures (right).

Outline

Estimation procedure

Estimation in 5 steps

1. Mean-reverting terms κ and $\mu(\cdot)$ $\longrightarrow$ CLSE
2. σ_T $\longrightarrow$ MLE
3. Dependence parameter λ $\longrightarrow$ Observed covariance
4. NIG parameters of L^X $\longrightarrow$ CLSE on CF
5. Goodness of fit of Model (ETM) $\longrightarrow$ χ^2-test goodness of fit

1. Estimation of κ and $\mu(\cdot)$

We estimate κ and $\mu(\cdot)$ through Conditional Least Square (CLS), Klimko and Nelson (1978):

$$\min_{\kappa,\alpha,\beta} \sum_{i=0}^{N-1} \left(X_{(i+1)\Delta} - \mathbb{E}[X_{(i+1)\Delta}|X_{i\Delta}]\right)^2,$$

is given, if $\hat{\eta}_2 \in (0,1)$, by

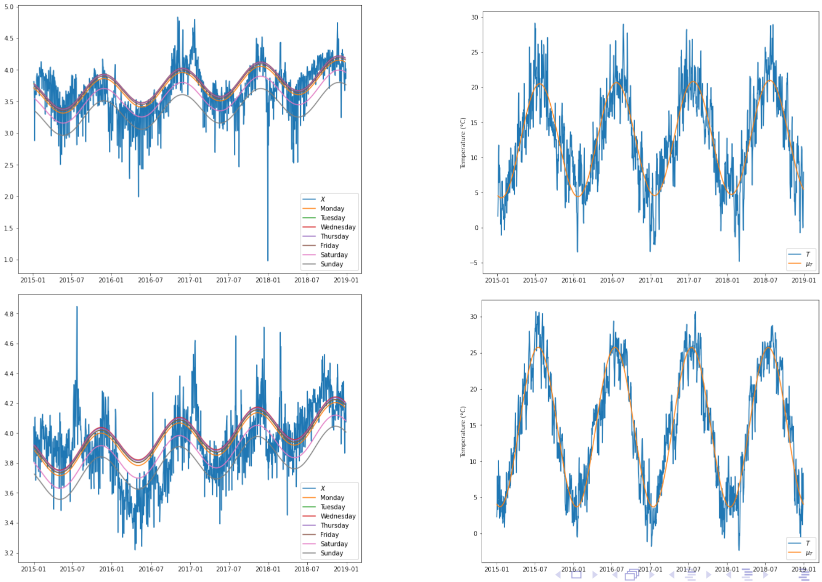
$$\begin{cases} \hat{\kappa}_X & = -\ln \hat{\eta}_2 \\ \hat{\beta}_0^X & = \frac{\hat{\eta}_1}{1-\hat{\eta}_2} \\ \hat{\alpha}_1^X & = \frac{\hat{\eta}_3(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})+\hat{\eta}_4\sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})^2+\sin^2(\xi\Delta)} \\ \hat{\beta}_1^X & = \frac{\hat{\eta}_4(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})-\hat{\eta}_3\sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})^2+\sin^2(\xi\Delta)} \\ \hat{\alpha}_j^{X,DoW} & = \frac{1}{1-e^{-7\hat{\kappa}_X\Delta}}\sum_{k=0}^{6}(\hat{\eta}_{j+k}^{DoW} - \hat{\beta}_0)e^{-(6-k)\hat{\kappa}_X\Delta}, \end{cases}$$

where

$$\hat{\eta} = \left(\sum_{i=0}^{N-1} \Xi_{i\Delta}\Xi_{i\Delta}^\top\right)^{-1}\left(\sum_{i=0}^{N-1} \Xi_{i\Delta}X_{(i+1)\Delta}\right),$$

with $\Xi_{i\Delta} = (i\Delta, X_{i\Delta}, \sin(\xi i\Delta), \cos(\xi i\Delta), (\mathbf{1}_{\{DoW(i\Delta)=j\}})_{0\le j\le 6}) \in \mathbb{R}^4 \times \{0,1\}^7$.

1. Estimation of κ and $\mu(\cdot)$



2. Estimation of σ_T

Let consider the integral of dynamics $(T_t)_{t\geq 0}$ from Model (ETM), $\Delta > 0$ and $\tilde{T}_t = T_t - \mu_T(t)$:

$$\tilde{T}_{t+\Delta} = e^{-\kappa_T \Delta}\tilde{T}_t + \sigma_T \underbrace{\int_t^{t+\Delta} e^{-\kappa_T(t-u)} dW_u^T}_{\sim\mathcal{N}\left(m^T\sqrt{\frac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}}, \sigma_T^2 \frac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}\right)}$$

$\Longrightarrow$ Easy to estimate through Maximum Likelihood Estimation (MLE).

Results

Market	$\hat{m}^T$	$\hat{\sigma}_T^2$
France	10^{-15}	2.413
North Italy	10^{-16}	1.846

Table: Parameter estimation through the maximum likelihood estimation for dynamic of temperature normally distributed for Paris and Milan temperature.

3. Estimation of the dependence parameter of λ

Estimation of λ

From Model (ETM),

$$\hat{\lambda} = \frac{\hat{\kappa}_X + \hat{\kappa}_T}{\hat{\sigma}_T^2 (1 - e^{-(\hat{\kappa}_X + \hat{\kappa}_T)\Delta})} \widehat{Cov}$$

Results

Market	$\hat{\lambda}$
France	-0.007
North Italy	-0.002

Table: Estimated λ of Model (ETM) for France and North Italy.

However, there is a dependency on quantile based contingency tables

197	162	126
163	160	162
124	163	197

165	173	149
176	147	163
145	166	175

Table: Observed frequencies by couple tercile for French (left) and Italian (right) coupled data.

3. Estimation of the dependence parameter of λ

Estimation of λ

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$$\hat{\lambda} = \frac{\hat{\kappa}_X + \hat{\kappa}_T}{\hat{\sigma}_T^2 (1 - e^{-(\hat{\kappa}_X + \hat{\kappa}_T)\Delta})} \widehat{Cov}$$

Results

Market	$\hat{\lambda}$	$\hat{\lambda}_{JFM}$	$\hat{\lambda}_{JJA}$
France	-0.007	-0.0175	0.0119
North Italy	-0.002	-0.0136	0.008

Table: Estimated λ of Model (ETM) for France and North Italy.

However, there is a dependency on quantile based contingency tables

197	162	126
163	160	162
124	163	197

165	173	149
176	147	163
145	166	175

Table: Observed frequencies by couple tercile for French (left) and Italian (right) coupled data.

4. Estimation of the NIG parameters for $(X_t)_{t\geq 0}$

We apply CLS estimation applied to the (conditional) characteristic function:

$$\text{minimize: } \sum_u \sum_{t=0}^{N-1} \left| e^{iu(\tilde{X}_{t+\Delta} - e^{-\kappa_X \Delta}\tilde{X}_t)} - e^{-\frac{1}{2}\lambda^2 \sigma_T^2 \frac{1-e^{-2\kappa_X \Delta}}{2\kappa_X} u^2} \varphi(u;\Delta) \right|^2, \quad (1)$$

where φ the characteristic function of $\left(\int_t^{t+\Delta} e^{-\kappa_X (t+\Delta-v)} dL_v^X \right)$

$$\varphi(u;\Delta) = \exp\left(ium^X \frac{1-e^{-\kappa_X \Delta}}{\kappa_X} + \delta\gamma^X \Delta - \delta^X \int_t^{t+\Delta} \sqrt{(\alpha^X)^2 - (\beta + iue^{-\kappa_X (t+\Delta-v)})^2} dv \right) \quad (2)$$

Results

9.43% and 4.34% of the standard deviation of the random term of the log energy spot price is explained by the temperature component for France and North Italy respectively.

5. Goodness of fit: χ^2-test on residuals

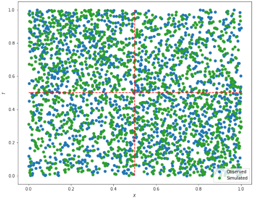

Figure: $p - value = 0.219$

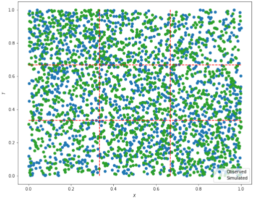

Figure: $p - value = 0.892$

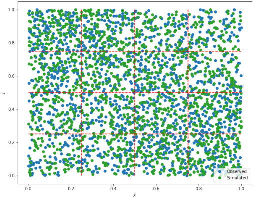

Figure: $p - value = 0.616$

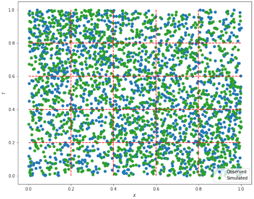

Figure: $p - value = 0.124$

Outline

Characterisation of quanto derivatives

Quanto payoff:

$$Payoff := \sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t)$$

In particular, for

	Futures	Swaps	Single-sided options	Double-sided options
$f_S(S_t)$	S_t	$(S_t - \bar{S})$	S_t	$(S_t - \bar{S})^+$
$f_T(T_t)$	T_t	$(\bar{T} - T_t)$	$(\bar{T} - T_t)^+$	$(\bar{T} - T_t)^+$

Application: $\bar{T} = 18°C$ (definition of HDD) and $\bar{S} = 50$ EUR/MWh.

Valuation of quanto derivatives

Explicit or semi-explicit formulas for:

$$\mathbb{E}\left(\sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t)\right)$$

Monte Carlo simulations (simulation scheme)

$$\begin{cases} X_{t+\Delta} & = \mu_X(t+\Delta) + e^{-\kappa_X \Delta}(X_t - \mu_X(t)) + \lambda\sigma_T\sqrt{\frac{1-e^{-2\kappa_X\Delta}}{2\kappa_X}}N_1 + e^{-\kappa_X\Delta/2}Z^X \\ T_{t+\Delta} & = \mu_T(t+\Delta) + e^{-\kappa_T \Delta}(T_t - \mu_T(t)) + \sigma_T\sqrt{\frac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}}(\rho N_1 + \sqrt{1-\rho^2}N_2), \end{cases}$$

where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,1)$ and $Z^X \sim NIG(\alpha^X, \beta^X, \delta^X, -\frac{\delta^X\beta^X}{\gamma^X})$

Pricing quanto derivatives

For double sided quanto options we suggest a first order Taylor 's expansion on λ for $t \in [t_1, t_2]$

$$\mathcal{Q}(t_1,t_2) = \sum_{t=t_1}^{t_2} \Bigg(\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) \times \Bigg(\Big(\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0)) \Big) \times$$

$$\Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big)$$

$$+ \frac{\sigma_T k_T(t-t_0)}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2} \Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big)^2 \Big) \Bigg)$$

$$-\Big(\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) + \bar{S}\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0}\right) \Big) \times$$

$$\sigma_T^2 k_{XT}(t-t_0)^2 \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{-\sigma_T k_T(t-t_0)} \Big) \Big) \lambda \Bigg) + o(\lambda)$$

where Φ is the cumulative distribution function of the standard Gaussian distribution, $k_T(\cdot)$, $k_X(\cdot)$ and $k_{XT}(\cdot)$ are defined in function of κ_X and κ_T.

Pricing quanto derivatives

For double sided quanto options we suggest a first order Taylor 's expansion on λ for $t \in [t_1, t_2]$

$$\begin{aligned}\mathcal{Q}(t_1,t_2) = \sum_{t=t_1}^{t_2} \Bigg(& \mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) \times \Bigg(\Big(\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0)) \Big) \times \\ & \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big) \\ & + \frac{\sigma_T k_T(t-t_0)}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2} \Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big)^2 \Big) \Bigg) \\ & - \Big(\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) + \bar{S}\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0}\right) \Big) \times \\ & \sigma_T^2 k_{XT}(t-t_0)^2 \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{-\sigma_T k_T(t-t_0)} \Big) \Big) \Big) \lambda \Bigg) + o(\lambda)\end{aligned} \tag{3}$$

where $\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0})$ is computed through Carr Madan formula and $\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0}\right)$ through Gil-Pelaez inversion.

Pricing quanto derivatives

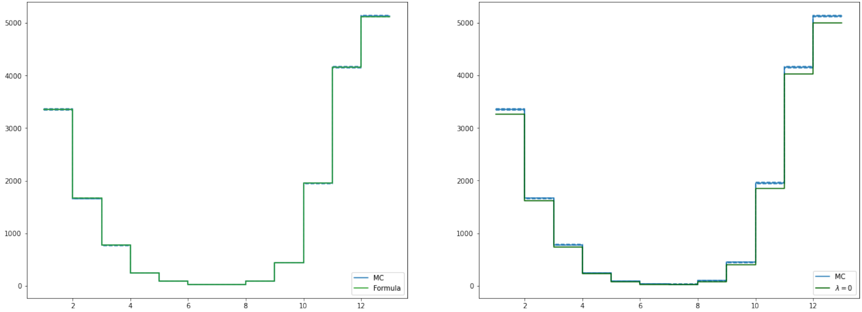

Figure: On the left, Quanto prices computed with 100,000 simulations Monte Carlo (blue) and semi-explicit (green) methods. On the right, Expansion without the first order term (i.e. formula with $\lambda = 0$). Each contract lasts a month of 2018. Time t_0 corresponds to 30 days ahead of the first day of the month, t_1 to the first day of the month and t_2 to the last day of the month.

Static hedging of quanto derivatives

Daily double-sided quantos

Under Model (ETM) and for $\Delta > 0$, let consider the portfolio:

$$\underbrace{(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+}_{\text{double-sided option quanto}} - \underbrace{d^0_{t,t+\Delta}}_{\text{cash}} - d^1_{t,t+\Delta}\underbrace{(\bar{T}-T_{t+\Delta})^+}_{\text{single HDD}} - d^2_{t,t+\Delta}\underbrace{(S_{t+\Delta}-\bar{S})^+}_{\text{call on spot}}$$

Static hedging of quanto derivatives

Daily double-sided quantos
Under Model (ETM) and for $\Delta > 0$, let consider the portfolio:

$$\mathbb{E}\Big[\Big((S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+ - d^0_{t,t+\Delta} - d^1_{t,t+\Delta}(\bar{T}-T_{t+\Delta})^+ - d^2_{t,t+\Delta}(S_{t+\Delta}-\bar{S})^+\Big)^2|\mathcal{F}_t\Big].$$

Static hedging of quanto derivatives

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$(d^0_{t,t+\Delta}, d^1_{t,t+\Delta}, d^2_{t,t+\Delta})$ minimising the above quadratic criterion is the unique solution of the linear system below:

$$\begin{bmatrix} 1 & \mathbb{E}[(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+|\mathcal{F}_t] \\ \mathbb{E}[(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((\bar{T}-T_{t+\Delta})^+)^2|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta}-\bar{S})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((S_{t+\Delta}-\bar{S})^+)^2|\mathcal{F}_t] \end{bmatrix} \begin{bmatrix} d^0_{t,t+\Delta} \\ d^1_{t,t+\Delta} \\ d^2_{t,t+\Delta} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta}-\bar{S})^+((\bar{T}-T_{t+\Delta})^+)^2|\mathcal{F}_t] \\ \mathbb{E}[((S_{t+\Delta}-\bar{S})^+)^2(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \end{bmatrix}$$

Static hedging of quanto derivatives

Daily double-sided quantos
Under Model (ETM) and for $\Delta > 0$, let consider the portfolio:

$$\mathbb{E}\Big[\Big((S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+ - d^0_{t,t+\Delta} - d^1_{t,t+\Delta}(\bar{T}-T_{t+\Delta})^+ - d^2_{t,t+\Delta}(S_{t+\Delta}-\bar{S})^+\Big)^2|\mathcal{F}_t\Big].$$

$(d^0_{t,t+\Delta}, d^1_{t,t+\Delta}, d^2_{t,t+\Delta})$ minimising the above quadratic criterion is the unique solution of the linear system below:

$$\begin{bmatrix} 1 & \mathbb{E}[(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+|\mathcal{F}_t] \\ \mathbb{E}[(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((\bar{T}-T_{t+\Delta})^+)^2|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta}-\bar{S})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((S_{t+\Delta}-\bar{S})^+)^2|\mathcal{F}_t] \end{bmatrix} \begin{bmatrix} d^0_{t,t+\Delta} \\ d^1_{t,t+\Delta} \\ d^2_{t,t+\Delta} \end{bmatrix} = \begin{bmatrix} \mathbb{E}[(S_{t+\Delta}-\bar{S})^+(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta}-\bar{S})^+((\bar{T}-T_{t+\Delta})^+)^2|\mathcal{F}_t] \\ \mathbb{E}[((S_{t+\Delta}-\bar{S})^+)^2(\bar{T}-T_{t+\Delta})^+|\mathcal{F}_t] \end{bmatrix}$$

Computed through first order Taylor expansion in λ, Carr Madan formula and Gil-Pelaez inversion formula.

Static hedging of quanto derivatives

Monthly double-sided quantos

Let know consider the monthly portfolio:

$$\sum_{i=1}^{31} (S_{t_1+i\Delta}-\bar{S})^+(\bar{T}-T_{t_1+i\Delta})^+ - d^0_{t_0,t_1+(i-1)\Delta} - d^1_{t_0,t_1+(i-1)\Delta}(\bar{T}-T_{t_1+i\Delta})^+ - d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta}-\bar{S})^+$$

and perform daily hedging as above to get:

	Without hedging	Hedging	Hedging $\lambda = 0$
January	$-3,358$	0.208	-93.072
May	-89.174	-0.113	-12.283

	Without hedging	Hedging	Hedging $\lambda = 0$
January	$2,197$	391	394
May	177	98	100

Table: Average (left) and standard deviation (right) of $\sum_{i=1}^{31} d^0_{t_0,t_1+(i-1)\Delta} + d^1_{t_0,t_1+(i-1)\Delta}(\bar{T}-T_{t_1+i\Delta})^+ + d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta}-\bar{S})^+ - (S_{t_1+i\Delta}-\bar{S})^+(\bar{T}-T_{t_1+i\Delta})^+$ for portfolio optimisation starting on 1st January 2018 and 1st May 2018 (for t_1), with $t_0 = t_1 - 30$ and lasting the whole month.

Static hedging of quanto derivatives

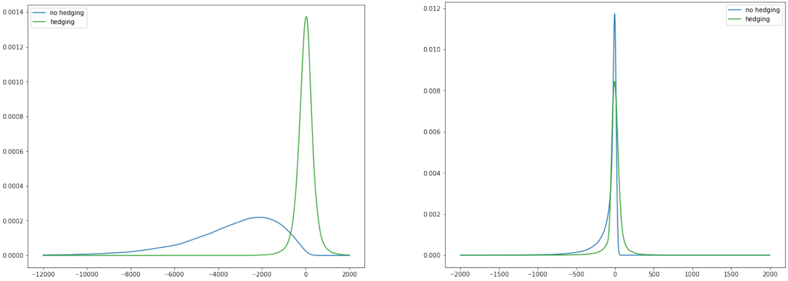

Figure: Empirical density of $\sum_{i=1}^{31} -(S_{t_1+i\Delta} - \bar{S})^+(\bar{T} - T_{t_1+i\Delta})^+$ (blue) and $\sum_{i=1}^{31} d^0_{t_0,t_1+(i-1)\Delta} + d^1_{t_0,t_1+(i-1)\Delta}(\bar{T} - T_{t_1+i\Delta})^+ + d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta} - \bar{S})^+ - (S_{t_1+i\Delta} - \bar{S})^+(\bar{T} - T_{t_1+i\Delta})^+$ (green) for portfolio optimisation starting on 1st January 2018 (for t_1 on the left) and 1st May 2018 (for t_1 on the right), with $t_0 = t_1 - 30$ and lasting the whole month.

Outline

Summary

- Develop a combined model for daily average temperature and electricity price.

  $\Longrightarrow$ Enables to understand risk related to coupled options.
- Overpass estimation challenges thanks to MLE and CLSE on characteristic function.
- Obtain explicit and semi-explicit formulas for futures, swap, single-sided and double-sided options.
- Show risk hedging capacity of single-sided ($\mathcal{E}$-HDD) and double-sided quanto options.

Thank you for your attention!