Loan selection for Collateralized Loan Obligation

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SMEs' financing needs

- SMEs have a key role in the economy: 90% of businesses and more than 50% of employment worldwide.
- Covid-19 and inflation: increase in financial vulnerability and cost of funds (OECD, 2023).
- About half of the SMEs have unmet credit needs (World Bank, 2017).
- Because private banks reduced their risk tolerance due to economic uncertainty.
- European Commission has launched several financing programmes for SMEs.
- Goal: reduce both interest rates and collateral requirements and increase financing volumes.

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Securitization of SME loans

Process of pooling different loans and selling their related cash flows to third party investors.

- Credit risk diversification.
- Allows financial institutions to transfer credit risk to investors willing to face those risks in exchange of excess return.
- Banks keep fees but relieve regulatory capital (which can be reinjected in the economy).
- Without this procedure, banks would remain dangerously exposed to the local housing and employment markets.
- Investors can reach a higher level of diversification and enhance their risks allocation.

Securitization at European Investment Fund

- Guarantee against the first credit losses (decreases the risk faced by investors and thus the cost of funds of SMEs).
- Cost associated with this guarantee can be seen as an injection of money aiming to stimulate some economic sectors.
- Useful to maximize the economic impact of each euro invested.
- Can focus on specific types of loans to boost high-priority sectors (e.g., cybersecurity or sustainable activities).
- Which loans to select for the sake of maximizing the economic impact?

Challenges

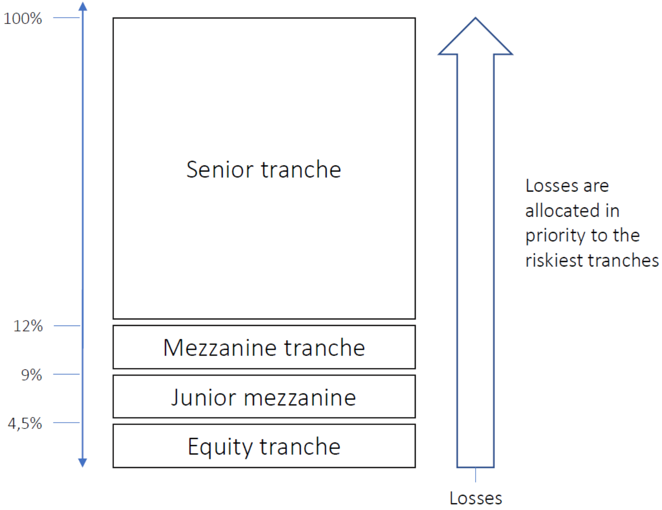
Assume we have N different loans and we need to assign a binary weight (selection vs exclusion) to each loan:

- Mixed-Integer non-linear optimization problem: NP-hard.
- N is typically large in practice (e.g. between 1.000 and 100.000 loans).
- Objective function is usually impacted by extreme events (large losses arising with low probability).
- This would induce heavy numerical procedures such as Monte Carlo simulations.

Contributions

1. How to formulate the objective function?
   - Point of view of an investor;
   - Point of view of an originator.
2. How to find an optimal solution? Use an approximate problem that:
   - Reduces the dimensionality of the problem;
   - Relaxes the binary constraint on the weight;
   - Tackles the non-linearity issue.

Collateralized Loan Obligation (CLO)



Investor objective function

$$\begin{aligned} \min_{\mathbf{w}} \quad & AL_{sen}(\mathbf{w}) \\ \text{s.t.} \quad & \sum w_i N_i \geq \underline{N} \\ & w_i \in \{0,1\} \quad \forall i \end{aligned}$$

- w_i is a binary weight for loan i,
- N_i is the principal outstanding of loan i,
- $\underline{N}$ is a lower bound for the total collateral,
- AL_{sen} is the Annualized Loss of the senior tranche: $AL_{sen}(\mathbf{w}) = \frac{EL_{sen}(\mathbf{w})}{WAL_{sen}(\mathbf{w})}$,
- EL_{sen} is the Expected Loss of the senior tranche,
- WAL_{sen} is the Weighted Average Life of the senior tranche.

Originator objective function

Ratio between the cost of releasing the senior tranche and the capital released:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{(D_{sen}-A_{sen})\left(\alpha+\beta\frac{EL_{sen}(\mathbf{w})}{WAL_{sen}(\mathbf{w})}\right)}{K_P(\mathbf{w})-(D_{jun}-A_{jun})K_{jun}(\mathbf{w})} \\ \text{s.t.} \quad & \sum w_i N_i \geq \underline{N} \\ & w_i \in \{0,1\} \quad \forall i \end{aligned}$$

- K_P is the capital requirement of the portfolio: $K_P = \frac{\sum_i w_i K_i N_i}{\sum_i w_i N_i}$,
- K_{jun} is the capital requirement of the junior tranche (non-linear function of K_P),
- A and D are respectively the attachment and detachment points of each tranche.

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Linearization of the Expected Loss

$$EL = \mathbb{E}\left[\min\left(\max\left(\frac{L_P - A}{D - A}; 0\right); 1\right)\right]$$

- L_P is the loss of the portfolio
- Large Homogeneous Portfolio approximation to model L_P
- Slack variables for min-max
- Gaussian Quadrature for the integral due to the expectation

Large Homogeneous Portfolio

Assume that the probability of default p_i of firm i is the probability that its assets Y_i fall below a threshold h_i:

$$p_i = \mathbb{P}(Y_i < h_i)$$

Assume Y_i is driven by two standard normal i.i.d. variables: a common random variable M and a firm-specific idiosyncratic random variable ϵ_i:

$$Y_i = \sqrt{\rho_i} M + \sqrt{1 - \rho_i} \epsilon_i$$

The sum of Gaussian variables is also Gaussian (*Vasicek, 2002*):

$$h_i = \Phi^{-1}(p_i)$$

Defaults are independent conditionally on the common factor M:

$$p_{i|M} = \mathbb{P}\left(\epsilon_i < \frac{h_i - \sqrt{\rho_i} M}{\sqrt{1 - \rho_i}}\right) = \Phi\left(\frac{h_i - \sqrt{\rho_i} M}{\sqrt{1 - \rho_i}}\right)$$

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Large Homogeneous Portfolio (cont'd)

Gordy (2003) proves that if the portfolio is sufficiently granular:

$$L_P \to \mathbb{E}[L_P \mid M]$$

The conditional loss of the portfolio is:

$$\mathbb{E}[L_P \mid M] = \frac{\sum_i w_i N_i \Phi\left(\frac{h_i - \sqrt{\rho_i} M}{\sqrt{1-\rho_i}}\right) LGD_i}{\sum_i w_i N_i}$$

The expected loss of tranche j can be approximated:

$$EL_j \approx \int_{-\infty}^{\infty} \min\left(\max\left(\frac{\mathbb{E}[L_P \mid M] - A_j}{D_j - A_j}; 0\right); 1\right) \Phi(M) dM$$

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Clustering

Optimization at cluster level instead of loan level:

- Low impact to choose between two similar loans (*Sirignano et al., 2016*);
- Dimensionality reduction;
- Binary decision variable replaced with a continuous one; the proportion we assign in a cluster;
- Select the loans with the smallest distance to centroid in priority;
- K-means.

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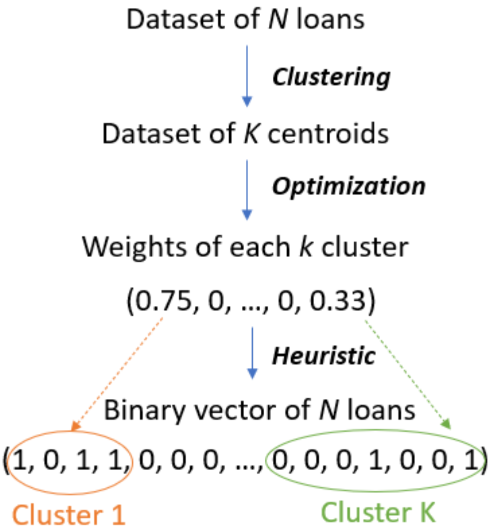
Clustering: illustration



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Benchmark

	Naive	Linear	Clustering	Linear + Clustering
Annualized Loss	3.256%	2.986%	3.035%	2.739%
Computation time	3953	3126	214	67

Table 1: Results for the investor problem

	Naive	Linear	Clustering	Linear + Clustering
Capital ratio	3.676%	3.637%	3.651%	3.506%
Computation time (s)	4590	3651	253	94

Table 2: Results for the originator problem

Key takeaways

- Investor's point of view: minimize Annualized Loss.
- Originator's point of view: minimize cost of capital release.
- Clustering to relax binary constraint and reduce dimensionality.
- Linear approximation of the main variables.
- Outperforms naive program both in terms of optimal solution and computation time.