Combinatorial Auctions with Illiquid Assets and Fairness Considerations: The Case of Blockchain Trade-Intent Auctions

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Blockchain is *permissionless & adversarial* ⇒ exchanging assets on chain *efficiently and safely* is challenging

- Anyone can create a marketplace for exchanging blockchain-based assets
- The same two assets may be exchanged in a dozen different markets (each with its mechanism, fees, liquidity, ...), incuding via intermediate assets
- Traders are routinely exploited via **sandwich attacks**
  - an attacker observes the victim's trade in the public mempool
  - front run the victim with a transaction in the same direction
  - back-runs the victim with a transaction in the opposite direction
  - the attacker buys low and sell high (at the expense of the victim)

The solution: Trade-Intent Auctions

- Each trader submits a **trade intent**: a trader specifies a sell and a buy token, a sell amount, and a limit price (a minimum they are willing to receive), *without specifying an execution path*.
- Specialized entities called **solvers** compete for the right to execute the traders' intents ⇒ the solver providing the "best price" wins.
- USD 7.6 B traded via trade intents in March 2024 (on Ethereum alone).

Trade-Intent Auctions in Practice

Specialization: some solvers may work with private market makers, others may optimize access to publicly available liquidity, others may specialize in niche tokens, ...

Complementarities: gas savings when multiple trades are executed together; coincidence of wants (direct p2p trading), including in intermediate legs of an execution.

Three main protocols / two mechanisms

- CoW Swap ⇒ batch auction: a single solver wins all trades in a batch; solvers' bids are evaluated using an oracle price
- Uniswap and 1inch ⇒ order-by-order Dutch (descending) auction

This research project: mechanism design of trade intent auctions

- A theoretical model to study different designs of trade-intent auctions
- Complementarities $\Rightarrow$ the design is combinatorial
- *Main challenge*: there is a notional market price for each asset, but exchanging it (at the time horizon of the auction) is subject to frictions and fees (i.e., tokens are *illiquid* within the auction)
  - $\Rightarrow$ sharing the efficiency from batching is subject to frictions/fees (like NTU models in cooperative game theory)
  - $\Rightarrow$ **fairness concerns**: batchign trades to generate additional efficiencies may not be beneficial for everybody

Example: why fairness considerations emerge

- Two traders, one wants to buy ETH and the other wants to buy DODGE
- Two solvers, one proposes a great deal for the first trader but a mediocre deal for the second trader, the second a great deal for the second trader but a mediocre deal for the first trader.
- When evaluated at the market price, the first solver's proposal is better.
- **Problem**: it is not possible to re-allocate tokens between traders at the notional market price.
- **Fairness**: which solution do you choose?

The model

Traders and solvers

Two traders, 1 and 2

1. sells 1 unit of token A for token B, their utility is $u_1 = x$ (x amount of token B received)
2. sells 1 unit of token C for token D, their utility is $u_2 = y$ (y amount of token D received)

Two solvers, 1 and 2

- if a solver is matched with no order, then it produces no tokens.
- if solver $i \in \{1,2\}$ is matched exclusively with order 1, it produces $\beta_i > 0$ units of token B and zero units of token D,
- if solver $i \in \{1,2\}$ is matched exclusively with order 2, it produces $\delta_i > 0$ units of token D and zero units of token B,
- if solver $i \in \{1,2\}$ is matched with both orders, it produces $g_i \cdot \beta_i$ units of token B and $g_i \cdot \delta_i$ units of token D, for $g_i \geq 1$.

Traders and solvers

- **Feasibility constraint**: solvers cannot return to the traders more than what they produced.
  - They cannot purchase additional tokens after the outcome of the auction is determined and they don't have an inventory.
- **Solvers' payoff**: the value of the tokens produced not returned to the traders, evaluated at the notional market prices
- δ_i β_i g_i are drawn at the beginning of the game, iid across solvers, and are private information.

The mechanism

- First stage: solvers bid on the individual trades
- Second stage: solvers bid on the entire batch (the two combined trades)
- The result of the first stage is unobservable before the second stage (ongoing work, the observable case)

Problem: not all mechanism are feasible. For example, for VCG and all pay mechanisms, for all non-trivial bids by one solver, there are bids by the other solver such that the first solver violates the feasibility constraint.

Assumption (to guarantee the feasibility of the mechanism)

- First price auction in the second stage: If the winner is determined in the second stage (i.e., batching), then the winner delivers its bid
- Two *simultaneous standard auctions* in the first stage (Gentry et al, 2019)

The mechanism's objective

- **Fairness**: a second-stage bid wins if and only if it delivers more to both traders relative to the outcome of the first-stage simultaneous standard auctions.
- if both second-stage bids are "fair", chose the one with higher market value

Sketch of the solution

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Benchmark: a single-trade auction

- Revenue equivalence theorem holds ⇒ First-price and second-price auction deliver the same expected utility to the traders
- If the auction is in second price, solvers bid their true value (i.e., what they can produce); if they are in first price, solvers shade their bids.

Remove the second bidding stage: two simultaneous auctions

- anticipating that it may also win the other trade, a solver may want to bid higher than in the benchmark (related to the *exposure problem*)
- The feasibility constraint prevents solvers from bidding higher than what they can produce
- ⇒ if the auctions are in second price, bids are like in the benchmark; if the auctions are in first price, bids are higher than in the benchmark.

Sketch of the solution

Fair combinatorial auction with **first-price** auctions in the first stage

In the first stage, bidders bid higher than is the benchmark in the first stage to disqualify the opponent's second-stage bid as unfair

- particular type of discontinuity: solvers may win or lose orders as a function of their bids without changing the ranking of the different outcomes in terms of market value

Fair combinatorial auction with **second-price** auctions in the first stage

- There is an equilibrium in which, in the first stage, bidders bid as in the benchmark
- There is an equilibrium in which, in the first stage, bidders bid the minimum amount (i.e., they don't bid)

Relevant literature (highly incomplete)

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- Trade intent market: Chitra et al. (2024)
- Simultaneous standard auctions: Gentry et al. (2019)
- First-price combinatorial auctions: Cantillon and Pesendorfer (2023)