A Deep Solver for BSDEs with Jumps

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Mathematical framework

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a filtered probability space satisfying the usual assumptions.

- $W = (W_t)_{t \in [0,T]}$: $\mathbb{R}^d$-valued standard Brownian motion;
- N: Poisson random measure with associated Lévy measure ν.
- $\tilde{N}$: compensated random measure

$$\tilde{N}(\mathrm{d}t, \mathrm{d}z) := N(\mathrm{d}t, \mathrm{d}z) - \nu(\mathrm{d}z)\mathrm{d}t.$$

We consider the following SDE in $\mathbb{R}^d$:

$$X_s = x + \int_t^s b(X_{r-})\, \mathrm{d}r + \int_t^s \sigma(X_{r-})\, \mathrm{d}W_r + \int_t^s \int_{\mathbb{R}^d} \Gamma(X_{r-}, z)\, \tilde{N}(\mathrm{d}r, \mathrm{d}z).$$

The vector fields b, σ, Γ are measurable functions such that:

- The functions b and σ are Lipschitz continuous;
- The function Γ satisfies, for some constant $K > 0$

$$|\Gamma(x,z)| \leq K(1 \wedge |z|), \quad (x,z) \in \mathbb{R}^d \times \mathbb{R}^d$$

$$\left|\Gamma(x,z) - \Gamma(x',z)\right| \leq K\left|x - x'\right|(1 \wedge |z|), \quad (x,z), (x',z) \in \mathbb{R}^d \times \mathbb{R}^d.$$

Mathematical framework

We aim at solving the following FBSDE:

$$
\begin{aligned}
Y_t =& g(X_T) + \int_t^T f\left(r, X_{r-}, Y_{r-}, Z_r, \int_{\mathbb{R}^d} U(r,z)\nu(\mathrm{d}z)\right)\mathrm{d}r - \int_t^T Z_r \mathrm{d}W_r \\
&- \int_t^T \int_{\mathbb{R}^d} U(r,z)\tilde{N}(\mathrm{d}r, \mathrm{d}z),
\end{aligned}
$$

where

- the function $f : [0,T] \times \mathbb{R}^d \times \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ satisfies

$$
\begin{aligned}
&\left| f(t,x,y,z,u) - f\left(t,x',y',z',u'\right)\right| \\
&\quad \le K\left(\left|x-x'\right| + \left|y-y'\right| + \left|z-z'\right| + \left|u-u'\right|\right)
\end{aligned}
$$

  for all $(t,x,y,z,u), (t,x',y',z',u')$, uniformly in t.
- the function $g : \mathbb{R}^d \to \mathbb{R}$ is measurable and satisfies

$$
\left|g(x) - g\left(x'\right)\right| \le K\left(\left|x-x'\right|\right), x, x' \in \mathbb{R}^d.
$$

Under these assumptions there exists a unique solution $(X^{t,x}, Y^{t,x}, Z^{t,x}, U^{t,x})$ of the FBSDE.

Mathematical framework

Consider the following PIDE

$$
\begin{aligned}
&-u_t(t,x) - \mathscr{L}u(t,x) \\
&\quad - f\left(t,x,u(t,x),u_x(t,x)\sigma(x),\mathscr{J}u(t,x)\right) = 0, \quad (t,x) \in [0,T) \times \mathbb{R}^d \\
&u(T,x) = g(x), \quad x \in \mathbb{R}^d,
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
\mathscr{L}u(t,x) &= \langle b(x), u_x(t,x)\rangle + \frac{1}{2}\langle \sigma(x)u_{xx}(t,x), \sigma(x)\rangle \\
&\quad + \int_{\mathbb{R}^d} \left(u(t,x+\Gamma(x,z)) - u(t,x) - \langle \Gamma(x,z), u_x(t,x)\rangle\right)\nu(dz) \\
\mathscr{J}u(t,x) &= \int_{\mathbb{R}^d} (u(t,x+\Gamma(x,z)) - u(t,x))\nu(dz).
\end{aligned}
$$

Mathematical framework

The link between FBSDEs and PIDEs is established via the Feynman-Kac formula. The following result holds:

Theorem (Delong, Theorem 4.2.1)

Let $u \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}^d, \mathbb{R})$ satisfy the PIDE above and the linear growth conditions

$$|u(t,x)| \leq K(1+|x|), \quad |u_x(t,x)| \leq K(1+|x|), \quad (t,x) \in [0,T] \times \mathbb{R}^d,$$

then

$$\begin{aligned} Y_s^{t,x} &= u\left(s, X_s^{t,x}\right), \quad t \leq s \leq T \\ Z_s^{t,x} &= u_x\left(s, X_{s-}^{t,x}\right)\sigma\left(X_{s-}^{t,x}\right), \quad t \leq s \leq T \\ U^{t,x}(s,z) &= u\left(s, X_{s-}^{t,x} + \Gamma\left(X_{s-}^{t,x}, z\right)\right) - u\left(s, X_{s-}^{t,x}\right), \quad t \leq s \leq T, z \in \mathbb{R}^d \end{aligned}$$

Mathematical framework

A Deep Solver for BSDEs with Jumps

Alessandro Gnoatto

For any $(t, x) \in [0, T] \times \mathbb{R}^d$ consider the following stochastic optimal control problem:

$$\underset{\substack{y, \\ Z = (Z_s)_{s\in[t,T]}, \\ U(s,\cdot) = (U(s,\cdot))_{s\in[t,T]}}}{\text{minimise}} \qquad \mathbb{E}\left[\left| g\left(X_T\right) - Y_T \right|^2 \middle| \mathcal{F}_t \right]$$

under the constraints

$$X_s = x + \int_t^s b(X_{r-})\mathrm{d}r + \int_t^s \sigma(X_{r-})\mathrm{d}W_r + \int_t^s \int_{\mathbb{R}^d} \Gamma(X_{r-}, z)\tilde{N}(\mathrm{d}r, \mathrm{d}z)$$

$$\begin{aligned} Y_s = y - \int_t^s f(r, X_{r-}, Y_{r-}, Z_r, \textstyle\int_{\mathbb{R}^d} U(r,z)\nu(\mathrm{d}z))\mathrm{d}r + \int_t^s Z_r \mathrm{d}W_r \\ + \int_t^s \int_{\mathbb{R}^d} U(r,z)\tilde{N}(\mathrm{d}r, \mathrm{d}z). \end{aligned}$$

A discretized version of this optimal control problem is at the basis of the **Deep BSDE solver** developed by E-Han-Jentzen ('17) in absence of jumps.

No jumps case: the Deep BSDE solver by E-Han-Jentzen ('17)

Consider the FBSDE:

$$\begin{cases} \mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t & X_0 = x \\ -\mathrm{d}Y_t = f\left(t, X_t, Y_t, Z_t\right)\mathrm{d}t - Z_t \mathrm{d}W_t & Y_T = g(X_T) \end{cases}$$

- Euler-Maruyama discretization ($\Delta t = \frac{T}{M}$, $M > 0$):

$$\begin{cases} X_{n+1} = X_n + b(X_n)\Delta t + \sigma(X_n)\Delta W_n, & X_0 = x \\ Y^{y,Z}_{n+1} = Y^{y,Z}_n - f(t_n, X_n, Y^{y,Z}_n, {\color{red}Z_n})\Delta t + {\color{red}Z_n}\Delta W_n, & Y^{y,Z}_0 = {\color{red}y}. \end{cases}$$

- Approximation of the control Z by an ANN (parametrized by ρ):

$$\begin{cases} X_{n+1} = X_n + b(X_n)\Delta t + \sigma(X_n)\Delta W_n, & X_0 = x \\ Y^{y,\rho}_{n+1} = Y^{y,\rho}_n - f(t_n, X_n, Y^{y,\rho}_n, {\color{red}\mathcal{Z}^\rho_n(X_n)})\Delta t + {\color{red}\mathcal{Z}^\rho_n(X_n)}\Delta W_n, & Y^{y,\rho}_0 = {\color{red}y}. \end{cases}$$

- Optimization:

$$\underset{{\color{red}y,\ \rho}}{\text{minimise}}\ \mathbb{E}\left[\left|g(X_M) - Y_M^{{\color{red}y,\rho}}\right|^2\right].$$

Presence of jumps: finite activity case

Let us assume that $\nu(\mathbb{R}^d) < \infty$.
We consider the following time-discretization of the dynamics:

$$\begin{aligned} X_{n+1} &= X_n + b(X_n)\Delta t + \sigma(X_n)\Delta W_n \\ &\quad + \sum_{i=N([0,t_n])+1}^{N([0,t_{n+1}])} \Gamma(X_n, z_i) - \Delta t \int_{\mathbb{R}^d} \Gamma(X_n, z)\nu(\mathrm{d}z), \quad x \in \mathbb{R}^d. \end{aligned}$$

$$\begin{aligned} Y_{n+1} =& Y_n - f\Big(t_n, X_n, Y_n, u_x(t_n, X_n)\sigma(X_n), \int_{\mathbb{R}^d} \left[u\left(t_n, X_n + \Gamma\left(X_n, z\right)\right) - u\left(t_n, X_n\right)\right] \nu(\mathrm{d}z)\Big)\Delta t \\ &+ u_x(t_n, X_n)\sigma(X_n)\Delta W_n \\ &+ \sum_{i=N([0,t_n])+1}^{N([0,t_{n+1}])} [u(t_n, X_n + \Gamma(X_n, z_i)) - u(t_n, X_n)] \\ &- \Delta t \int_{\mathbb{R}^d} u\left(t_n, X_n + \Gamma(X_n, z)\right) - u(t_n, X_n)\nu(\mathrm{d}z). \end{aligned}$$

where $N([0,t])$ is the number of jumps occurring in $[0,t]$, each of an $\mathbb{R}^d$-valued size z_i.

Presence of jumps: finite activity case

We now introduce ANN approximations.
A feedforward neural network is a function defined via the composition

$$x \longmapsto \mathcal{A}_{\mathcal{L}} \circ \varrho \circ \mathcal{A}_{\mathcal{L}-1} \circ \ldots \circ \varrho \circ \mathcal{A}_1(x),$$

where all $\mathcal{A}_\ell$, $\ell = 1, \ldots, \mathcal{L}$, are affine transformations of the form $\mathcal{A}_\ell(x) := \mathcal{W}_\ell x + \beta_\ell$, $\ell = 1, \ldots, \mathcal{L}$, where $\mathcal{W}_\ell$ and β_ℓ are matrices and vectors of suitable size, and the function ϱ, is a (nonlinear) activation function applied component-wise to vectors.

We need two families of feed-forward ANNs:

- $\mathcal{U}_n^\rho$ for parametrizing the function $u(t_n, \cdot)$, for $n = 0, \ldots, M$. Parameters: ρ;
- $\mathcal{V}_n^\eta$ for parametrizing the compensator term:

$$\int_{\mathbb{R}^d} u\left(t_n, X_n + \Gamma(X_n, z)\right) - u\left(t_n, X_n\right) \nu(\mathrm{d}z),$$

  for $n = 1, \ldots, M$. Parameters: η.

Presence of jumps: finite activity case

Remark. The introduction of the ANN $\mathcal{V}_n^\eta$ allows us to avoid nested MC simulations that would naturally arise if one tries to approximate the term

$$\int_{\mathbb{R}^d} \mathcal{U}_n^\rho\left(X_n + \Gamma(X_n, z)\right) - \mathcal{U}_n^\rho\left(X_n\right) \nu(\mathrm{d}z).$$

The backward dynamics becomes:

$$\begin{aligned} Y_{n+1}^{\rho,\eta} = & Y_n^{\rho,\eta} - f\Big(t_n, X_n, Y_n^{\rho,\eta}, (\mathcal{U}_n^\rho)_x \sigma(X_n), \mathcal{V}_n^\eta(X_n)\Big)\Delta t \\ & + (\mathcal{U}_n^\rho)_x \sigma(X_n)\Delta W_n \\ & + \sum_{i=N([0,t_n])+1}^{N([0,t_{n+1}])} [\mathcal{U}_n^\rho(X_n + \Gamma(X_n, z_i)) - \mathcal{U}_n^\rho(X_n)] \\ & - \Delta t \mathcal{V}_n^\eta(X_n). \end{aligned}$$

Presence of jumps: finite activity case

We recall that, under our assumptions, the stochastic integral with respect to the compensated Poisson random measure is a square integrable martingale, i.e.

$$\mathbb{E}\left[\int_{t_n}^{t_{n+1}}\int_{\mathbb{R}^d} u\left(r, X_{r-}+\Gamma(X_{r\_}, z)\right)-u\left(r, X_{r-}\right)\tilde{N}(\mathrm{d}r,\mathrm{d}z)\middle|\mathcal{F}_{t_n}\right]=0$$

Let us recall that the conditional expectation of a random variable $\mathscr{X}\in L^2(\Omega,\mathcal{F})$ with respect to $\mathcal{F}_{t_n}\subset\mathcal{F}$ satisfies

$$\mathbb{E}\left[\mathscr{X}\,|\,\mathcal{F}_{t_n}\right]=\underset{\widetilde{\mathscr{X}}\in L^2(\Omega,\mathcal{F}_{t_n})}{\operatorname{argmin}}\ \mathbb{E}\left[(\mathscr{X}-\widetilde{\mathscr{X}})^2\right]$$

Presence of jumps: finite activity case

so that

$$0 = \underset{\widetilde{\mathscr{X}} \in L^2(\Omega, \mathcal{F}_{t_n})}{\operatorname{argmin}} \mathbb{E}\left[\left(\int_{t_n}^{t_{n+1}} \int_{\mathbb{R}^d} u\left(r, X_{r-} + \Gamma(X_{r\_}, z)\right) - u\left(r, X_{r-}\right) \tilde{N}(\mathrm{d}r, \mathrm{d}z) - \widetilde{\mathscr{X}} \right)^2 \right].$$

This inspires the introduction of a further penalty term to be minimized during the training phase: at each time t_n, we would like to minimize the penalty term

$$\mathbb{E}\left[\left(\sum_{i=N([0,t_n])+1}^{N([0,t_{n+1}])} [\mathcal{U}_n^{\rho}(X_n + \Gamma(X_n, z_i)) - \mathcal{U}_n^{\rho}(X_n)] - \Delta t \mathcal{N}_{t_n}^{\nu, \rho_\nu}(X_{t_n}) \right)^2 \right].$$

Presence of jumps: finite activity case

We then consider the following optimization problem:

$$
\begin{aligned}
&\underset{\rho,\eta}{\text{minimize}}\, \mathbb{E}\left[\left(g\left(X_M\right)-Y_M^{\rho,\eta}\right)^2\right] \\
&\quad+\sum_{n=0}^{M-1}\mathbb{E}\left[\left(\sum_{i=N([0,t_n])+1}^{N([0,t_{n+1}])}\left[\mathcal{U}_n^{\rho}\left(X_n+\Gamma(X_n,z_i)\right)-\mathcal{U}_n^{\rho}\left(X_n\right)\right]-\Delta t\mathcal{V}_n^{\eta}\left(X_n\right)\right)^2\right]
\end{aligned}
$$

Numerical results

Test 1:
Consider the following pure jump process in $\mathbb{R}$

$$\frac{\mathrm{d}X_t}{X_{t-}} = \int_{\mathbb{R}} (e^z - 1)(N - \nu)(\mathrm{d}z, \mathrm{d}t)\ ,$$

where $\nu(\mathrm{d}z) = \lambda\varphi(z)\mathrm{d}z$ with $\lambda > 0$ and $\varphi(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$.

Let

$$u(t, X_t) = \mathbb{E}\left[X_T | \mathcal{F}_t\right].$$

Numerical results

By Ito's formula one can show that u is the solution of the following PIDE:

$$\begin{cases} u_t + \int_{\mathbb{R}} [u(t, xe^z) - u(t,x) - x(e^z - 1)u_x]\nu(\mathrm{d}z) = 0, \\ u(T,x) = x \end{cases}$$

Therefore, $Y_t := u(t, X_t)$ clearly solves the BSDE:

$$Y_t = X_T - \int_t^T \int_{\mathbb{R}^d} U(r,z)\tilde{N}(\mathrm{d}r, \mathrm{d}z).$$

However one also has

$$\begin{aligned} Y_t &= \mathbb{E}\left[e^{i\phi \ln X_T} \middle| \mathcal{F}_t \right] \Big|_{\phi=-1} \\ &= \exp\left\{ i\phi \ln X_t - \lambda i\phi \left(e^{\mu + \frac{1}{2}\sigma^2} - 1 \right)(T-t) + \lambda \left(e^{i\phi\mu - \frac{1}{2}u^2\sigma^2} - 1 \right)(T-t) \right\} \\ &= X_t \end{aligned}$$

Numerical results

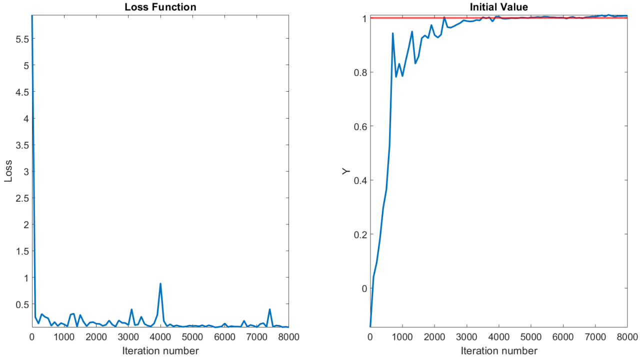

Figure: The loss function value (left) and the fitted initial value X_0 (right) increasing the iteration number for the pure jump expectation example. In red the theoretical value $Y_0^{\text{exact}} = X_0 = 1$. Error 0.76%. CPU 894 s.

Numerical results

Test 2:

Let X be the price of a stock described by the following dynamics

$$\frac{\mathrm{d}X_t}{X_{t-}} = r\mathrm{d}t + \sigma \mathrm{d}W_t^{\mathbb{Q}} + \int_{\mathbb{R}} (e^z - 1)(N - \nu)(\mathrm{d}z, \mathrm{d}t)\,, \quad X_0 = x_0 \in \mathbb{R}.$$

The value of a European call option is given by

$$Y_t = \mathbb{E}^{\mathbb{Q}}\left[e^{-r(T-t)}(X_T - k)^+ | \mathcal{F}_t \right].$$

Numerical results

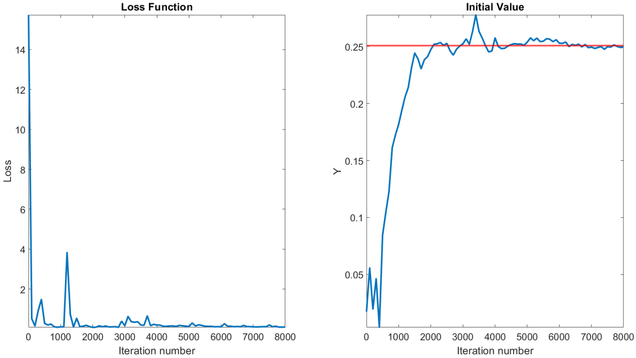

Figure: The loss function value (left) and the fitted initial value Y_0 (right) increasing the iteration number for the call option example. In red the Monte Carlo fitted value $Y_0^{mc} = 0.251$. Error 0.46%. CPU 927 s.

Numerical results

Test 3:
We consider the case of several underlying assets $(X^1; \cdots ; X^d)$:

$$\frac{\mathrm{d}X_t^i}{X_{t-}^i} = r^i \mathrm{d}t + \sigma^i \mathrm{d}W_t^{\mathbb{Q},i} + \int_{\mathbb{R}} (e^z - 1)(N - \nu)(\mathrm{d}z, \mathrm{d}t) , \quad X_0 = x_0^i \in \mathbb{R}^d, \quad i = 1, \cdots$$

where $W^{\mathbb{Q}} = (W^{\mathbb{Q},1}, \cdots , W^{\mathbb{Q},2})$ is a standard Brownian motion in $\mathbb{R}^d$.
We set $d = 100$.
The value of a European basket call option is given by

$$Y_t = \mathbb{E}^{\mathbb{Q}}\left[e^{-r(T-t)}\left(\sum_{i=1}^{d} X_T^i - k\right)^+\right].$$

Preliminary numerical results

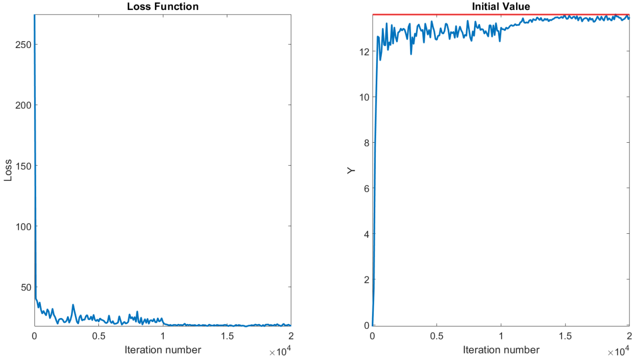

Figure: The loss function value (left) and the fitted initial value Y_0 (right) increasing the iteration number for the basket call option example of dimension 100. In red the Monte Carlo fitted value $Y_0^{mc} = 13.607$. Error 0.95%. CPU 4438 s.

Presence of jumps: infinite activity case

To deal with the possible presence of infinite jumps we introduce a finite activity approximating forward process.
We assume we can factorize the jump term in two components, the first one corresponding to the small jumps, and the second one to the big jumps.
Let $\epsilon \in (0,1)$. We define

$$\nu^{\epsilon}(\mathrm{d}z) := 1_{\{|z|>\epsilon\}}\nu(\mathrm{d}z)$$
$$\nu_{\epsilon}(\mathrm{d}z) := 1_{\{|z|\leq\epsilon\}}\nu(\mathrm{d}z)$$

and assume that $\nu = \nu^{\epsilon} + \nu_{\epsilon}$ with $\int_{\mathbb{R}^d} \|z\|^2 \nu_{\epsilon}(\mathrm{d}z) < \infty$ and $\nu^{\epsilon}\left(\mathbb{R}^d\right) < \infty$.
This factorization of the Lévy measure means that we are assuming that we can write the Poisson Random Measure N as

$$N = N_{\epsilon} + N^{\epsilon}$$

with N_{ϵ} and N^{ϵ} having the compensator ν_{ϵ} and ν^{ϵ}, respectively.
We also assume that the coefficient Γ is of the form

$$\Gamma(x,z) := \gamma(x)z$$

for some function $\gamma : \mathbb{R}^d \mapsto \mathbb{R}^{d\times d}$.

Presence of jumps: infinite activity case

Let us define

$$\Sigma_\epsilon := \int_{\mathbb{R}^d} zz^\top \nu_\epsilon(\mathrm{d}z) = \int_{|z|<\epsilon} zz^\top \nu(\mathrm{d}zx).$$

We approximate the forward process X by X^ϵ given by

$$X_s^\epsilon = x + \int_t^s b\left(X_{r-}^\epsilon\right)\mathrm{d}r + \int_t^s \sigma\left(X_{r-}^\epsilon\right)\mathrm{d}W_r + \gamma\left(X_{r-}^\epsilon\right)\sqrt{\Sigma_\epsilon}\mathrm{d}W_r$$
$$+ \int_t^s \int_{\mathbb{R}^d} \gamma\left(X_{r-}^\epsilon\right) z\tilde{N}^\epsilon(\mathrm{d}r, \mathrm{d}z), \quad x \in \mathbb{R}^d,$$

where we set $\tilde{N}^\epsilon := N^\epsilon(\mathrm{d}r, \mathrm{d}z) - \nu^\epsilon(\mathrm{d}z)\mathrm{d}r$.

Presence of jumps: infinite activity case

Recall the original BSDE:

$$
\begin{aligned}
Y_t = & g(X_T) + \int_t^T f\left(s, X_{r-}, Y_{r-}, Z_r, \int_{\mathbb{R}^d} U(r,z)\nu(\mathrm{d}z)\right)\mathrm{d}r - \int_t^T Z_r^\top \mathrm{d}W_r \\
& - \int_t^T \int_{\mathbb{R}^d} U(r,z)\tilde{N}(\mathrm{d}r,\mathrm{d}z).
\end{aligned}
$$

We substitute the forward process with the approximation X^ϵ:

$$
\begin{aligned}
Y_t^\epsilon = & g(X_T^\epsilon) + \int_t^T f\left(s, X_{r-}^\epsilon, Y_{r-}^\epsilon, Z_r^\epsilon, \int_{\mathbb{R}^d} U^\epsilon(r,z)\nu(\mathrm{d}z)\right)\mathrm{d}r - \int_t^T Z_r^{\epsilon,\top} \mathrm{d}W_r \\
& - \int_t^T \int_{\mathbb{R}^d} U^\epsilon(r,z)\tilde{N}(\mathrm{d}r,\mathrm{d}z).
\end{aligned}
$$

We substitute the Poisson Random measure with the approximated one:

$$
\begin{aligned}
Y_t^{\epsilon\epsilon} = & g(X_T^\epsilon) + \int_t^T f\left(s, X_{r-}^\epsilon, Y_{r-}^{\epsilon\epsilon}, Z_r^{\epsilon\epsilon}, \int_{\mathbb{R}^d} U^{\epsilon\epsilon}(r,z)\nu^\epsilon(\mathrm{d}z)\right)\mathrm{d}r \\
& - \int_t^T Z_r^{\epsilon\epsilon,\top} \mathrm{d}W_r - \int_t^T \int_{\mathbb{R}^d} U^{\epsilon\epsilon}(r,z)\tilde{N}^\epsilon(\mathrm{d}r,\mathrm{d}z).
\end{aligned}
$$

CGMY Process

The CGMY is a pure jump Lévy process L with characteristic triplet $(0, 0, \nu(\mathrm{d}z))$, where the Lévy measure is given by

$$\nu(\mathrm{d}z) = C\frac{e^{-G|z|}}{|z|^{1+Y}}1_{\{z<0\}}\mathrm{d}z + C\frac{e^{-Mz}}{z^{1+Y}}1_{\{z>0\}}\mathrm{d}z.$$

We can write $L_t = L_t^+ - L_t^-$ where

- L^+has triplet $(0, 0, \nu^+(\mathrm{d}z))$, where $\nu^+(\mathrm{d}z) := C\frac{e^{-Mz}}{z^{1+Y}}1_{\{z>0\}}\mathrm{d}z$;
- L^-has triplet $(0, 0, \nu^-(\mathrm{d}z))$, where $\nu^-(\mathrm{d}z) := C\frac{e^{-Gz}}{z^{1+Y}}1_{\{z>0\}}\mathrm{d}z$.

Approximating Jump-Diffusion

We approximate L via

$$L^{\epsilon}_{t_{n+1}} = L^{\epsilon}_{t_n} + \sigma_\epsilon \Delta W_{n+1} + \sum_{j=1}^{N^+_{n+1}} \Delta L^+_{n+1,j} - b_{\epsilon^+}\Delta_{n+1} - \sum_{j=1}^{N^-_{n+1}} \Delta L^-_{n+1,j} + b_{\epsilon^-}\Delta_{n+1}$$

where

$$N^+_{n+1} \sim \mathcal{POISS}\left(\lambda^+_\epsilon, \Delta_{n+1}\right),$$
$$N^-_{n+1} \sim \mathcal{POISS}\left(\lambda^-_\epsilon, \Delta_{n+1}\right)$$
$$\Delta L^+_{n+1,j} \sim f_{\epsilon^+}, \quad \Delta L^-_{n+1,j} \sim f_{\epsilon^-}$$

$$f_{\epsilon^+} = \frac{1}{\lambda^+_\epsilon} C \frac{e^{-Mz}}{z^{1+Y}} 1_{\{z>\epsilon\}}$$

and similarly for f_{ϵ^-}.

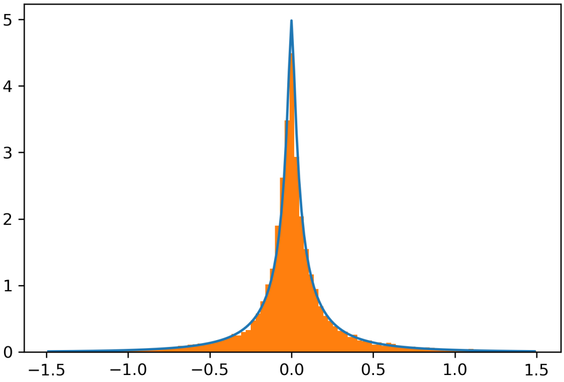

Figure: Histogram of the empirical (Monte Carlo) density (orange bars) and density obtained by FFT inversion of the characteristic function of the true process (blue line). Parameters: $C = 0.1$; $G = 1.4$; $M = 1.3$; $Y = 0.5$; $\epsilon = 0.0001$

A European call: $g(S_T) = \left(S_T - k\right)^+$

We set $S_0 = 1$, $k = 0.9$, $r = 0.04$. 100 time steps.

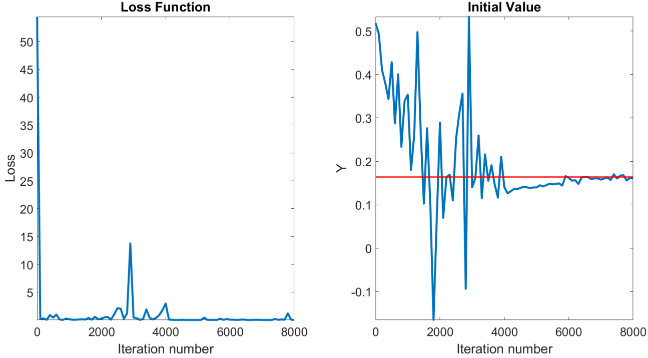

Figure: The loss function value (plot on the left) and the fitted initial value Y_0 (plot on the right) increasing the iteration number for the call option example. In red the Monte Carlo approximation $Y_0^{mc} = 0.164$

Conclusions

- Generalization of the Deep BSDE solver in the presence of jumps.
- Treatment of the infinite activity case.

A. Gnoatto, M. Patacca A. Picarelli. *A Deep Solver for BSDEs with jumps*, Submitted.

Currently, we are generalizing the analysis of Han and Long in order to derive a priori and a posteriori error bounds for the algorithm in the decoupled case. This is a joint work with Athena Picarelli (Univ. Verona) and Katharina Oberpriller (LMU Munich).

A. Gnoatto, A. Picarelli, K. Oberpriller. *Convergence of the Deep BSDE solver with jumps*, in preparation.

We are hiring! Tenure Track Position

The University of Verona is seeking to fill **one Tenure track position in Mathematical Finance**

- Fixed term position for 6 years
- The researcher who obtains the Italian National Scientific Habilitation can ask for the conversion into a tenured Associate Professor position **starting from the third year**.
- Teaching duties: 90 hours per year in both our campuses in Verona and Vicenza. Willingness to learn Italian is expected.

For any question feel free to get in touch

`alessandro.gnoatto@univr.it`

We are hiring! 5 PhD Positions

The University of Verona is seeking to fill **5 PhD positions in Mathematical Finance**

- Duration of 4 years.
- Financial support for an international research stay of at least 6 months and for the attendance of conferences.
- Intensive training program during the first year with international guest lecturers.

For any question feel free to get in touch

`alessandro.gnoatto@univr.it`

AMaMeF 2025 in Verona

The University of Verona will host the 12th AMaMeF (Advanced Mathematical Methods for Finance) conference in 2025.

- The program will consist of plenary lectures, invited and contributed sessions, and posters, addressing a full range of topics in mathematical finance.
- Call for papers later this year.

Conference Website

`https://sites.google.com/view/amamef2025/home`